# NORMALITY OF THE THUE–MORSE SEQUENCE ALONG PIATETSKI-SHAPIRO SEQUENCES, II

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Abstract. We prove that the Thue–Morse sequence $\mathbf{t}$ along subsequences indexed by $\lfloor n^c \rfloor$ is normal, where $1 < c < 3/2$. That is, for $c$ in this range and for each $\omega \in \{0,1\}^L$, where $L \geq 1$, the set of occurrences of $\omega$ as a factor (contiguous finite subsequence) of the sequence $n \mapsto \mathbf{t}_{\lfloor n^c \rfloor}$ has asymptotic density $2^{-L}$. This is an improvement over a recent result by the second author, which handles the case $1 < c < 4/3$.

In particular, this result shows that for $1 < c < 3/2$ the sequence $n \mapsto \mathbf{t}_{\lfloor n^c \rfloor}$ attains both of its values with asymptotic density $1/2$, which improves on the bound $c < 1.4$ obtained by Mauduit and Rivat (who obtained this bound in the more general setting of *q-multiplicative functions*, however) and on the bound $c \leq 1.42$ obtained by the second author.

In the course of proving the main theorem, we show that $2/3$ is an *admissible level of distribution* for the Thue–Morse sequence, that is, it satisfies a Bombieri–Vinogradov type theorem for each exponent $\eta < 2/3$. This improves on a result by Fouvry and Mauduit, who obtained the exponent $0.5924$. Moreover, the underlying theorem implies that every finite word $\omega \in \{0,1\}^L$ is contained as an arithmetic subsequence of $\mathbf{t}$.