Abstract

Let $s_q$ be the sum-of-digits function in base $q$, $q \geq 2$. If $t$ is a positive integer, we denote by $t^R$ the unique integer that is obtained from $t$ by reversing the order of the digits of the proper representation of $t$ in base $q$. In this work we prove that for all $\alpha \in \mathbb{R}$ and all positive integers $t$ the correlation measure

$$\gamma(\alpha, t) = \lim_{x \to \infty} \frac{1}{x} \sum_{n < x} e^{2\pi i \alpha (s_q(n+t) - s_q(n))}$$

satisfies $\gamma(\alpha, t) = \gamma(\alpha, t^R)$. From this we deduce that for all integers $d$ the sets \{ $n \in \mathbb{N} : s_q(n + t) - s_q(n) = d$ \} and \{ $n \in \mathbb{N} : s_q(n + t^R) - s_q(n) = d$ \} have the same asymptotic density. The proof involves methods coming from the study of $q$-additive functions, linear algebra, and analytic number theory.