

ALL MIXED IDENTITIES ARE SINGULAR IN GROUPS WITHOUT ALGEBRAICITY

MICHAEL PINSKER
TU WIEN

ALGEBRA SEMINAR
TU WIEN
2.10.2026

FWF Austrian
Science Fund

T5948



European Research Council
Established by the European Commission

SyG PococOP
GA 101071674

MY FIRST THEOREM IN ALGEBRA

MICHAEL PINSKER
TU WIEN

ALGEBRA SEMINAR
TU WIEN
2. 10. 2026

FWF Austrian
Science Fund

T5948



European Research Council
Established by the European Commission

SyG PococOP
GA 101071674

LOGIC COLLOQUIUM 2025

ITAY KAPLAN: ARE ALL MIXED IDENTITIES OF $\text{Aut}(\mathbb{Q}, <)$ SINGULAR?

LOGIC COLLOQUIUM 2025

ITAY KAPLAN: ARE ALL MIXED IDENTITIES OF $\text{Aut}(Q, <)$ SINGULAR?



EQUATION $w(\underbrace{x_1, \dots, x_n}_{\text{VARIABLES}}; \underbrace{y_1, \dots, y_r}_{\in \text{Aut}(Q, <)}) = 1$

TRUE FOR ALL $x_1, \dots, x_n$

LOGIC COLLOQUIUM 2025

ITAY KAPLAN: ARE ALL MIXED IDENTITIES OF $\text{Aut}(\mathbb{Q}, <)$ SINGULAR?



EQUATION $w(\underbrace{x_1, \dots, x_n}_{\text{VARIABLES}}; \underbrace{y_1, \dots, y_k}_{\in \text{Aut}(\mathbb{Q}, <)}) = 1$
TRUE FOR ALL $x_1, \dots, x_n$

EXAMPLE: • $x_1 x_2 x_1^{-1} x_2^{-1} = 1$

LOGIC COLLOQUIUM 2025

ITAY KAPLAN: ARE ALL MIXED IDENTITIES OF $\text{Aut}(\mathbb{Q}, <)$ SINGULAR?



EQUATION $w(\underbrace{x_1, \dots, x_n}_{\text{VARIABLES}}; \underbrace{y_1, \dots, y_k}_{\in \text{Aut}(\mathbb{Q}, <)}) = 1$
TRUE FOR ALL $x_1, \dots, x_n$

EXAMPLE: • $x_1 x_2 x_1^{-1} x_2^{-1} = 1$ **X**

LOGIC COLLOQUIUM 2025

ITAY KAPLAN: ARE ALL MIXED IDENTITIES OF $\text{Aut}(\mathbb{Q}, <)$ SINGULAR?



EQUATION $w(\underbrace{x_1, \dots, x_n}_{\text{VARIABLES}}; \underbrace{y_1, \dots, y_k}_{\in \text{Aut}(\mathbb{Q}, <)}) = 1$

TRUE FOR ALL $x_1, \dots, x_n$

- EXAMPLE:
- $x_1 x_2 x_1^{-1} x_2^{-1} = 1$ **X**
 - $x_1^{-1} x_1 = 1$

LOGIC COLLOQUIUM 2025

ITAY KAPLAN: ARE ALL MIXED IDENTITIES OF $\text{Aut}(\mathbb{Q}, <)$ SINGULAR?



EQUATION $w(\underbrace{x_1, \dots, x_n}_{\text{VARIABLES}}; \underbrace{y_1, \dots, y_r}_{\in \text{Aut}(\mathbb{Q}, <)}) = 1$

TRUE FOR ALL $x_1, \dots, x_n$

- EXAMPLE:
- $x_1 x_2 x_1^{-1} x_2^{-1} = 1$ ✗
 - $x_1^{-1} x_1 = 1$ ✓ TRIVIAL

LOGIC COLLOQUIUM 2025

ITAY KAPLAN: ARE ALL MIXED IDENTITIES OF $\text{Aut}(Q, <)$ SINGULAR?



EQUATION $w(\underbrace{x_1, \dots, x_n}_{\text{VARIABLES}}; \underbrace{y_1, \dots, y_k}_{\in \text{Aut}(Q, <)}) = 1$

TRUE FOR ALL $x_1, \dots, x_n$

EXAMPLE:

- $x_1 x_2 x_1^{-1} x_2^{-1} = 1$ ✗
- $x_1^{-1} x_1 = 1$ ✓ TRIVIAL

- $[[y_1^x, y_3], [y_2^x, y_2]] = 1$

WHERE

$$\text{Supp } y_1 < \text{Supp } y_2 < \text{Supp } y_3$$

LOGIC COLLOQUIUM 2025

ITAY KAPLAN: ARE ALL MIXED IDENTITIES OF $\text{Aut}(Q, <)$ SINGULAR?



EQUATION $w(\underbrace{x_1, \dots, x_n}_{\text{VARIABLES}}; \underbrace{y_1, \dots, y_k}_{\in \text{Aut}(Q, <)}) = 1$

TRUE FOR ALL $x_1, \dots, x_n$

EXAMPLE:

- $x_1 x_2 x_1^{-1} x_2^{-1} = 1$ ✗
- $x_1^{-1} x_1 = 1$ ✓ TRIVIAL
- $[[y_1^x, y_3], [y_2^x, y_2]] = 1$ ✓ SINGULAR

WHERE

$$\text{Supp } y_1 < \text{Supp } y_2 < \text{Supp } y_3$$

LOGIC COLLOQUIUM 2025

ITAY KAPLAN: ARE ALL MIXED IDENTITIES OF $\text{Aut}(Q, <)$ SINGULAR?

EQUATION

↙

$$w(\underbrace{x_1, \dots, x_n}_{\text{VARIABLES}}; \underbrace{y_1, \dots, y_k}_{\text{Aut}(Q, <)}) = 1$$

TRUE FOR ALL $x_1, \dots, x_n$

EXAMPLE:

- $x_1 x_2 x_1^{-1} x_2^{-1} = 1$ ✗
- $x_1^{-1} x_1 = 1$ ✓ TRIVIAL
- $[[y_1^x, y_3], [y_2^x, y_2]] = 1$ ✓ SINGULAR

WHERE

$$\text{Supp } y_1 < \text{Supp } y_2 < \text{Supp } y_3$$

„FORGETTING“
CONSTANTS
W BECOMES
TRIVIAL

E.G.

$$x_1 y_1^{-1} x_1^{-1} x_2 y_1^{-1} x_2^{-1} = 1$$

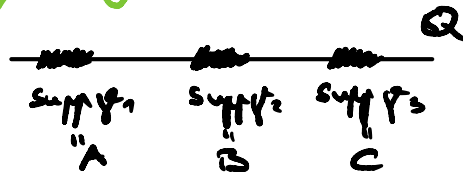
LOGIC COLLOQUIUM 2025

ITAY KAPLAN: ARE ALL MIXED IDENTITIES OF $\text{Aut}(Q, <)$ SINGULAR?

EQUATION

$w(\underbrace{x_1, \dots, x_n}_{\text{VARIABLES}}; \underbrace{y_1, \dots, y_k}_{\text{GAUT}(Q, <)}) = 1$

TRUE FOR ALL $x_1, \dots, x_n$



EXAMPLE:

- $x_1 x_2 x_1^{-1} x_2^{-1} = 1$ **X**
- $x_1^{-1} x_1 = 1$ **✓** TRIVIAL
- $[[\varphi_1^x, \varphi_3], [\varphi_2^x, \varphi_2]] = 1$ **✓** SINGULAR

WHERE

$$\text{supp } \varphi_1 < \text{supp } \varphi_2 < \text{supp } \varphi_3$$

LOGIC COLLOQUIUM 2025

ITAY KAPLAN: ARE ALL MIXED IDENTITIES OF $\text{Aut}(Q, <)$ SINGULAR?

EQUATION

$w(\underbrace{x_1, \dots, x_n}_{\text{VARIABLES}}; \underbrace{y_1, \dots, y_k}_{\in \text{Aut}(Q, <)}) = 1$

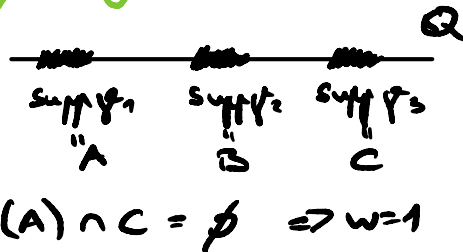
TRUE FOR ALL $x_1, \dots, x_n$

EXAMPLE:

- $x_1 x_2 x_1^{-1} x_2^{-1} = 1$ **X**
- $x_1^{-1} x_1 = 1$ **✓** TRIVIAL
- $[[y_1^x, y_3], [y_2^x, y_2]] = 1$ **✓** SINGULAR

WHERE

$$\text{Supp } y_1 < \text{Supp } y_2 < \text{Supp } y_3$$



LOGIC COLLOQUIUM 2025

ITAY KAPLAN: ARE ALL MIXED IDENTITIES OF $\text{Aut}(Q, <)$ SINGULAR?

EQUATION

$w(\underbrace{x_1, \dots, x_n}_{\text{VARIABLES}}; \underbrace{y_1, \dots, y_k}_{\in \text{Aut}(Q, <)}) = 1$

TRUE FOR ALL $x_1, \dots, x_n$

EXAMPLE:

- $x_1 x_2 x_1^{-1} x_2^{-1} = 1$ **X**
- $x_1^{-1} x_1 = 1$ **✓** TRIVIAL

- $[[y_1^x, y_3], [y_2^x, y_2]] = 1$ **✓** SINGULAR

WHERE

$$\text{Supp } y_1 < \text{Supp } y_2 < \text{Supp } y_3$$

$\overline{\text{---}} \quad \overline{\text{---}} \quad \overline{\text{---}} \quad Q$
 $\text{Supp } y_1 \quad \text{Supp } y_2 \quad \text{Supp } y_3$
"A" "B" "C"

- $x(A) \cap C = \emptyset \Rightarrow w=1$
- $x(A) \cap C \neq \emptyset$
 $\Rightarrow x(B) \cap B = \emptyset$
 $\Rightarrow w=1$ **□**

LOGIC COLLOQUIUM 2025

ITAY KAPLAN: ARE ALL MIXED IDENTITIES OF $\text{Aut}(Q, <)$ SINGULAR?



EQUATION $w(\underbrace{x_1, \dots, x_n}_{\text{VARIABLES}}; \underbrace{y_1, \dots, y_k}_{\in \text{Aut}(Q, <)}) = 1$

TRUE FOR ALL $x_1, \dots, x_n$

- EXAMPLE:
- $x_1 x_2 x_1^{-1} x_2^{-1} = 1$ ✗
 - $x_1^{-1} x_1 = 1$ ✓ TRIVIAL
 - $[[y_1^x, y_3], [y_2^x, y_2]] = 1$ ✓ SINGULAR

WHERE

$$\text{Supp } y_1 < \text{Supp } y_2 < \text{Supp } y_3$$

CONNECTIONS WITH ZARISKI TOPOLOGY GIVEN BY SOLUTION SETS

TOPOLOGIES ON ENDOMORPHISM MONOIDS OF GENERIC STRUCTURES

MICHAEL PINSKER
TU WIEN



European Research Council
Established by the European Commission

FWF Austrian
Science Fund

T5948

SyG PococOP
GA 101071674

MCW XXIX
2024

$G = (G, \cdot, 1, {}^{-1})$ GROUP

REGULAR MIXED IDENTITIES 3

$G = (G, \cdot, 1, {}^{-1})$ GROUP

REGULAR MIXED IDENTITIES ?



MIXED IDENTITY

$w(x_1 \dots x_n, g_1 \dots g_r) = 1$ HOLDS,

$u \in G, u \neq 1$

$\Rightarrow u w = 1$ NOT SOLVABLE

$G = (G, \cdot, 1, {}^{-1})$ GROUP

REGULAR MIXED IDENTITIES ?



MIXED IDENTITY

$w(x_1 \dots x_n, g_1 \dots g_n) = 1$ HOLDS,

$u \in G, u \neq 1$

$\Rightarrow u w = 1$ NOT SOLVABLE



G FINITE $\Rightarrow w!G| = 1$ HOLDS

$G = (G, \cdot, 1, {}^{-1})$ GROUP

REGULAR MIXED IDENTITIES ?



MIXED IDENTITY

$w(x_1 \dots x_n, g_1 \dots g_c) = 1$ HOLDS,

$u \in G, u \neq 1$

$\Rightarrow u w = 1$ NOT SOLVABLE



G FINITE $\Rightarrow w|G| = 1$ HOLDS

CONJECTURE (ULKATCHUKO, THOM '16)

w REGULAR $\Rightarrow \exists H \cong G$

$w = 1$ SOLVABLE IN H

G FINITE $\Rightarrow H$ FINITE

$G = \langle G, \cdot, 1, {}^{-1} \rangle$ GROUP

REGULAR MIXED IDENTITIES ?



MIXED IDENTITY

$w(x_1 \dots x_n, g_1 \dots g_e) = 1$ HOLDS,

$u \in G, u \neq 1$

$\Rightarrow u w = 1$ NOT SOLVABLE



G FINITE $\Rightarrow w|G| = 1$ HOLDS

CONJECTURE (ULKATCHUKO, THOM '16)

w REGULAR $\Rightarrow \exists H \cong G$

$w = 1$ SOLVABLE IN H

G FINITE $\Rightarrow H$ FINITE

ONE VARIABLE:

KERVARE-LAUDENBACH
CONJECTURE

$G = \langle G, \cdot, 1, {}^{-1} \rangle$ GROUP

REGULAR MIXED IDENTITIES ?



MIXED IDENTITY

$w(x_1 \dots x_n, g_1 \dots g_e) = 1$ HOLDS,

$u \in G, u \neq 1$

$\Rightarrow u w = 1$ NOT SOLVABLE



G FINITE $\Rightarrow w!G! = 1$ HOLDS

CONJECTURE (ULKATCHUKO, THOM '16)

w REGULAR $\Rightarrow \exists H \cong G$

$w = 1$ SOLVABLE IN H

G FINITE $\Rightarrow H$ FINITE

ONE VARIABLE:

KERVARE-LAUDENBACH
CONJECTURE

REGULARITY:

$$a^2 = 1, b^2 \neq 1$$

$$x a x^{-1} b^{-1} = 1$$

$G = \langle G, \cdot, 1, {}^{-1} \rangle$ GROUP

REGULAR MIXED IDENTITIES ?



MIXED IDENTITY

$w(x_1 \dots x_n, g_1 \dots g_r) = 1$ HOLDS,

$u \in G, u \neq 1$

$\Rightarrow u w = 1$ NOT SOLVABLE



G FINITE $\Rightarrow w \nmid |G| = 1$ HOLDS

CONJECTURE (ULKATCHUKO, THOM '16)

w REGULAR $\Rightarrow \exists H \cong G$

$w = 1$ SOLVABLE IN H

G FINITE $\Rightarrow H$ FINITE

ONE VARIABLE:

KERVARE-LAUDENBACH
CONJECTURE

PROVED FOR:

- G FINITE, ONE VARIABLE
(GERSTENHABER, ROTHBAUS '62)

$G = \langle G, \cdot, 1, {}^{-1} \rangle$ GROUP

REGULAR MIXED IDENTITIES?



MIXED IDENTITY

$w(x_1 \dots x_n, g_1 \dots g_c) = 1$ HOLDS,

$u \in G, u \neq 1$

$\Rightarrow u w = 1$ NOT SOLVABLE



G FINITE $\Rightarrow w|G| = 1$ HOLDS

CONJECTURE (ULKATCHUKO, THOM '16)

w REGULAR $\Rightarrow \exists H \cong G$

$w = 1$ SOLVABLE IN H

G FINITE $\Rightarrow H$ FINITE

ONE VARIABLE:

KERVARE-LAUDENBACH
CONJECTURE

PROVED FOR:

- G FINITE, ONE VARIABLE
(GERSTENHABER, ROTHBAUS '62)
- HYPERLINEAR G (SEE PESTOV '08)

$G = \langle G, \cdot, 1, {}^{-1} \rangle$ GROUP

REGULAR MIXED IDENTITIES?



MIXED IDENTITY

$w(k_1 \dots x_n, g_1 \dots g_e) = 1$ HOLDS,

$u \in G, u \neq 1$

$\Rightarrow u w = 1$ NOT SOLVABLE



G FINITE $\Rightarrow w|G| = 1$ HOLDS

CONJECTURE (ULKATCHUKO, THOM '16)

w REGULAR $\Rightarrow \exists H \cong G$

$w = 1$ SOLVABLE IN H

G FINITE $\Rightarrow H$ FINITE

ONE VARIABLE:

KERVARE-LAUDENBACH
CONJECTURE

PROVED FOR:

- G FINITE, ONE VARIABLE
(GERSTENHABER, ROTHBAUS '62)
- HYPERLINEAR G (SEE PESTOV '08)

THEOREM (ADELEKE, MOLLARD '94)

ALL $t(k_1 \dots x_n) = g$ SOLVABLE IN
 $\text{Aut}(G, <)$

$G = (G, \cdot, 1, {}^{-1})$ GROUP

REGULAR MIXED IDENTITIES ?



MIXED IDENTITY

$w(x_1 \dots x_n, g_1 \dots g_s) = 1$ HOLDS,

$u \in G, u \neq 1$

$\Rightarrow u w = 1$ NOT SOLVABLE



G FINITE $\Rightarrow w|G| = 1$ HOLDS

CONJECTURE (ULKATCHUKO, THOM '16)

w REGULAR $\Rightarrow \exists H \cong G$

$w = 1$ SOLVABLE IN H

G FINITE $\Rightarrow H$ FINITE

ONE VARIABLE:

KERVARE-LAUDENBACH
CONJECTURE

PROVED FOR:

- G FINITE, ONE VARIABLE
(GERSTENHABER, ROTHBAUS '62)
- HYPERLINEAR G (SEE PESTOV '08)

THEOREM (ADELEKE, MOLLARD '94)

ALL $f(x_1 \dots x_n) = g$ SOLVABLE IN
 $\text{Aut}(G, <)$

THEOREM (SCHNEIDER, THOM '22)

$\text{Sym}(N)$ NO REGULAR MIXED IDENTITIES

$G = \langle G, \cdot, 1, {}^{-1} \rangle$ GROUP

REGULAR MIXED IDENTITIES ?



MIXED IDENTITY

$w(k_1 \dots x_n, g_1 \dots g_r) = 1$ HOLDS,

$u \in G, u \neq 1$

$\Rightarrow u w = 1$ NOT SOLVABLE



G FINITE $\Rightarrow w \nmid |G| = 1$ HOLDS

CONJECTURE (ULKATCHUKO, THOM '16)

w REGULAR $\Rightarrow \exists H \cong G$

$w = 1$ SOLVABLE IN H

G FINITE $\Rightarrow H$ FINITE

ONE VARIABLE:

KERVARE-LAUDENBACH
CONJECTURE

PROVED FOR:

- G FINITE, ONE VARIABLE
(GERSTENHABER, ROTHBAUS '62)
- HYPERLINEAR G (SEE PESTOV '08)

THEOREM (ADELEKE, MOLLARD '94)

ALL $w(k_1 \dots x_n) = g$ SOLVABLE IN
 $\text{Aut}(G, <)$

THEOREM (SCHNEIDER, THOM '22)

$\text{Sym}(N)$ NO REGULAR MIXED IDENTITIES

THEOREM (GOLUBCHIK, MIKHAEV '82)

V VECTOR SPACE OVER INFINITE FIELD

$\Rightarrow GL(V)$ NO REGULAR MIXED IDENTITIES

"RICHNESS" OF $\mathcal{L}$ SHOULD AVOID
REGULAR IDENTITIES

"RICHNESS" OF G SHOULD AVOID
REGULAR IDENTITIES

X ... COUNTABLY INFINITE SET

$G \curvearrowright X$ OLIGOMORPHIC $\Leftrightarrow$

$\forall n \geq 1 \quad G \curvearrowright X^n$ FINITELY MANY ORBITS

"RICHNESS" OF G SHOULD AVOID
REGULAR IDENTITIES

$X \dots$ COUNTABLY INFINITE SET

$G \curvearrowright X$ OLIGOMORPHIC $\Leftrightarrow$
 $\forall n \geq 1 \ G \curvearrowright X^n$ FINITELY MANY ORBITS

THEOREM (MACPHERSON '65)

G OLIGOMORPHIC $\Rightarrow$ NO IDENTITIES
WITHOUT CONSTANTS

"RICHNESS" OF $\mathcal{L}$ SHOULD AVOID
REGULAR IDENTITIES

$X \dots$ COUNTABLY INFINITE SET

$\mathcal{G} \curvearrowright X$ OLIGOMORPHIC $\Leftrightarrow$

$\forall n \geq 1 \mathcal{G} \curvearrowright X^n$ FINITELY MANY ORBITS

THEOREM (MACPHERSON '65)

$\mathcal{L}$ OLIGOMORPHIC $\Rightarrow$ NO IDENTITIES
WITHOUT CONSTANTS

THEOREM AUTOMORPHISM GROUPS OF:

- RANDOM GRAPH ETENDJALABADI, CAO, LE MAÎTRE, MELERANT '21
- DIGRAPH DE LA NUEZ, GHADERNEZHAD '19
- POSET } BOBRSKY, THOM, SCHNIDER '24
- PERMUTATION }
- VECTOR SPACE OVER FINITE FIELD BRADFORD, THOM, SCHNEIDER
- HALL'S UNIVERSAL GROUP
- URSOHN SPACE (ETENDJALABADI, CAO, LE MAÎTRE, MELERANT '21)

NO REGULAR MIXED IDENTITIES!

"RICHNESS" OF $\mathcal{L}$ SHOULD AVOID
REGULAR IDENTITIES

$X \dots$ COUNTABLY INFINITE SET

$\mathcal{G} \curvearrowright X$ **OLIGOMORPHIC** $\Leftrightarrow$
 $\forall n \geq 1 \mathcal{G} \curvearrowright X^n$ FINITELY MANY ORBITS

THEOREM (MACPHERSON '65)

$\mathcal{G}$ OLIGOMORPHIC $\Rightarrow$ NO IDENTITIES
WITHOUT CONSTANTS

THEOREM AUTOMORPHISM GROUPS OF:

- RANDOM GRAPH ETENDJALABADI, CAO, LE MAÎTRE, MELCERAY '21
- DIGRAPH DE LA NÚEZ, GHADERNEZHAD '19
- POSET } BOJARSKY, THOM, SCHNEIDER '24
- PERMUTATION }
- VECTOR SPACE OVER FINITE FIELD BRADFORD, THOM, SCHNEIDER
- HALL'S UNIVERSAL GROUP
- URYSONN SPACE (ETENDJALABADI, CAO, LE MAÎTRE, MELCERAY '21)

NO REGULAR MIXED IDENTITIES!

CONJECTURE (BOJARSKY, SCHNEIDER, THOM '23)

$\mathcal{G}$ OLIGOMORPHIC

$\Rightarrow$ NO REGULAR MIXED IDENTITIES

"RICHNESS" OF $\mathcal{L}$ SHOULD AVOID
REGULAR IDENTITIES

$X \dots$ COUNTABLY INFINITE SET

$G \curvearrowright X$ **OLIGOMORPHIC** $\Leftrightarrow$
 $\forall n \geq 1 \quad G \curvearrowright X^n$ FINITELY MANY ORBITS

THEOREM (MACPHERSON '65)

G OLIGOMORPHIC $\Rightarrow$ NO IDENTITIES
WITHOUT CONSTANTS

THEOREM AUTOMORPHISM GROUPS OF:

- RANDOM GRAPH ETENDJALABADI, CAO, LE MAÎTRE, MELCERAY '21
- DIGRAPH DE LA NUEZ, GHADERNEZHAD '19
- POSET } BOJARSKY, THOM, SCHNEIDER '24
- PERMUTATION }
- VECTOR SPACE OVER FINITE FIELD BRADFORD, THOM, SCHNEIDER
- HALL'S UNIVERSAL GROUP
- URYSONN SPACE (ETENDJALABADI, CAO, LE MAÎTRE, MELCERAY '21)

NO REGULAR MIXED IDENTITIES!

CONJECTURE (BOJARSKY, SCHNEIDER, THOM '23)

G OLIGOMORPHIC

$\Rightarrow$ NO REGULAR MIXED IDENTITIES



$\text{Aut}(\mathbb{Q}, <) \text{ OPEN!}$

"RICHNESS" OF $\mathcal{L}$ SHOULD AVOID
REGULAR IDENTITIES

"RICHNESS" OF G SHOULD AVOID
REGULAR IDENTITIES

ALGEBRAIC CLOSURE WRT $G \curvearrowright X$

$F \subseteq X$ FINITE

$$\text{acl}(F) = \{x \in X \mid x \text{ HAS FINITE ORBIT UNDER } G_F\}$$

NO ALGEBRAICITY: $\forall F \text{acl}(F) = F$

"RICHNESS" OF G SHOULD AVOID
REGULAR IDENTITIES

ALGEBRAIC CLOSURE WRT $G \curvearrowright X$

$F \subseteq X$ FINITE

$$\text{acl}(F) = \{x \in X \mid x \text{ HAS FINITE ORBIT UNDER } G_F\}$$

NO ALGEBRAICITY: $\forall F \text{acl}(F) = F$

EXAMPLE

"RICHNESS" OF G SHOULD AVOID
REGULAR IDENTITIES

ALGEBRAIC CLOSURE WRT $G \curvearrowright X$

$F \subseteq X$ FINITE

$$\text{acl}(F) = \{x \in X \mid x \text{ HAS FINITE ORBIT UNDER } G_F\}$$

NO ALGEBRAICITY: $\forall F \text{ } \text{acl}(F) = F$

EXAMPLE

- $\text{Sym}(X)$

"RICHNESS" OF G SHOULD AVOID
REGULAR IDENTITIES

ALGEBRAIC CLOSURE WRT $G \curvearrowright X$

$F \subseteq X$ FINITE

$$\text{acl}(F) = \{x \in X \mid x \text{ HAS FINITE ORBIT UNDER } G_F\}$$

NO ALGEBRAICITY: $\forall F \text{ } \text{acl}(F) = F$

EXAMPLE

- $\text{Sym}(X)$ ✓

"RICHNESS" OF G SHOULD AVOID
REGULAR IDENTITIES

ALGEBRAIC CLOSURE WRT $G \curvearrowright X$

$F \subseteq X$ FINITE

$$\text{acl}(F) = \{x \in X \mid x \text{ HAS FINITE ORBIT UNDER } G_F\}$$

NO ALGEBRAICITY: $\forall F \text{ } \text{acl}(F) \neq F$

EXAMPLE

- $\text{Sym}(X)$ ✓
- $\text{Aut}(\mathbb{Q}, <)$

"RICHNESS" OF G SHOULD AVOID
REGULAR IDENTITIES

ALGEBRAIC CLOSURE WRT $G \curvearrowright X$

$F \subseteq X$ FINITE

$$\text{acl}(F) = \{x \in X \mid x \text{ HAS FINITE ORBIT UNDER } G_F\}$$

NO ALGEBRAICITY: $\forall F \text{ } \text{acl}(F) \neq F$

EXAMPLE

- $\text{Sym}(X)$ ✓
- $\text{Aut}(\mathbb{Q}, <)$ ✓

"RICHNESS" OF G SHOULD AVOID
REGULAR IDENTITIES

ALGEBRAIC CLOSURE WRT $G \curvearrowright X$

$F \subseteq X$ FINITE

$$\text{acl}(F) = \{x \in X \mid x \text{ HAS FINITE ORBIT UNDER } G_F\}$$

NO ALGEBRAICITY: $\forall F \text{ } \text{acl}(F) = F$

EXAMPLE

- $\text{Sym}(X)$ ✓
- $\text{Aut}(\mathbb{Q}, <)$ ✓
- $\text{Aut}(\text{random graph})$

"RICHNESS" OF G SHOULD AVOID
REGULAR IDENTITIES

ALGEBRAIC CLOSURE WRT $G \curvearrowright X$

$F \subseteq X$ FINITE

$$\text{acl}(F) = \{x \in X \mid x \text{ HAS FINITE ORBIT UNDER } G_F\}$$

NO ALGEBRAICITY: $\forall F \text{ } \text{acl}(F) = F$

EXAMPLE

- $\text{Sym}(X)$ ✓
- $\text{Aut}(\mathbb{Q}, <)$ ✓
- $\text{Aut}(\text{random graph})$ ✓

"RICHNESS" OF G SHOULD AVOID
REGULAR IDENTITIES

ALGEBRAIC CLOSURE WRT $G \curvearrowright X$

$F \subseteq X$ FINITE

$$\text{acl}(F) = \{x \in X \mid x \text{ HAS FINITE ORBIT UNDER } G_F\}$$

NO ALGEBRAICITY: $\forall F \text{ } \text{acl}(F) = F$

EXAMPLE

- $\text{Sym}(X)$ ✓
- $\text{Aut}(\mathbb{Q}, <)$ ✓
- $\text{Aut}(\text{random graph})$ ✓
- $\text{GL}(V)$

"RICHNESS" OF G SHOULD AVOID
REGULAR IDENTITIES

ALGEBRAIC CLOSURE WRT $G \curvearrowright X$

$F \subseteq X$ FINITE

$$\text{acl}(F) = \{x \in X \mid x \text{ HAS FINITE ORBIT UNDER } G_F\}$$

NO ALGEBRAICITY: $\forall F \text{ } \text{acl}(F) \neq F$

EXAMPLE

- $\text{Sym}(X)$ ✓
- $\text{Aut}(\mathbb{Q}, <)$ ✓
- $\text{Aut}(\text{random graph})$ ✓
- $\text{GL}(V)$ ✗

"RICHNESS" OF $\mathcal{L}$ SHOULD AVOID
REGULAR IDENTITIES

ALGEBRAIC CLOSURE

$F \subseteq X$ FINITE

$$\text{acl}(F) = \{x \in X \mid x \text{ HAS FINITE ORBIT UNDER } G_F\}$$

NO ALGEBRAICITY: $\forall F \text{ } \text{acl}(F) = F$

EXAMPLE

- $\text{Sym}(X)$ ✓
- $\text{Aut}(\mathbb{Q}, <)$ ✓
- $\text{Aut}(\text{random graph})$ ✓
- $\text{GL}(V)$ ✗



STRONG AMALGAMATION (SAP)

$\Rightarrow$

NO ALGEBRAICITY

"RICHNESS" OF G SHOULD AVOID
REGULAR IDENTITIES

ALGEBRAIC CLOSURE

$F \subseteq X$ FINITE

$ac(F) = \{x \in X \mid x \text{ HAS FINITE ORBIT UNDER } G_F\}$

NO ALGEBRAICITY: $\forall F \quad ac(F) = F$

EXAMPLE

- $Sym(X)$ ✓
- $Aut(Q, <)$ ✓
- $Aut(\text{random graph})$ ✓
- $GL(V)$ ✗



STRONG AMALGAMATION (SAP)

$\Rightarrow$

NO ALGEBRAICITY

CONJECTURE (BOODISLY, SCHNEIDER, THOM 23)

G OLIGOMORPHIC

$\Rightarrow$ NO REGULAR MIXED IDENTITIES



$Aut(Q, <)$ OPEN!

"RICHNESS" OF $\mathcal{L}$ SHOULD AVOID
REGULAR IDENTITIES

ALGEBRAIC CLOSURE

$F \subseteq X$ FINITE

$\text{acl}(F) = \{x \in X \mid x \text{ HAS FINITE ORBIT UNDER } G_F\}$

NO ALGEBRAICITY: $\forall F \text{ } \text{acl}(F) = F$

EXAMPLE

- $\text{Sym}(X)$ ✓
- $\text{Aut}(\mathbb{Q}, <)$ ✓
- $\text{Aut}(\text{random graph})$ ✓
- $\text{GL}(V)$ ✗

👁️ STRONG AMALGAMATION (SAP)

⇒

NO ALGEBRAICITY

CONJECTURE (BOODISLY, SCHNEIDER, THOM '23)

$\mathcal{L}$ OLIGOMORPHIC, NO ALGEBRAICITY

⇒ NO REGULAR MIXED IDENTITIES



$\text{Aut}(\mathbb{Q}, <)$ OPEN!

"RICHNESS" OF $\mathcal{L}$ SHOULD AVOID
REGULAR IDENTITIES

ALGEBRAIC CLOSURE

$F \subseteq X$ FINITE

$\text{acl}(F) = \{x \in X \mid x \text{ HAS FINITE ORBIT UNDER } G_F\}$

NO ALGEBRAICITY: $\forall F \text{ } \text{acl}(F) = F$

EXAMPLE

- $\text{Sym}(X)$ ✓
- $\text{Aut}(\mathbb{Q}, <)$ ✓
- $\text{Aut}(\text{random graph})$ ✓
- $\text{GL}(V)$ ✗

👁️ STRONG AMALGAMATION (SAP)

↳

NO ALGEBRAICITY

THEOREM (MARINON, P. '26)

$\mathcal{L}$ OLIGOMORPHIC, NO ALGEBRAICITY

⇒ NO REGULAR MIXED IDENTITIES

👁️ $\text{Aut}(\mathbb{Q}, <)$ ~~OPEN!~~

"RICHNESS" OF $\mathcal{L}$ SHOULD AVOID
REGULAR IDENTITIES

ALGEBRAIC CLOSURE

$F \subseteq X$ FINITE

$\text{acl}(F) = \{x \in X \mid x \text{ HAS FINITE ORBIT UNDER } G_F\}$

NO ALGEBRAICITY: $\forall F \text{ } \text{acl}(F) = F$

EXAMPLE

- $\text{Sym}(X)$ ✓
- $\text{Aut}(\mathbb{Q}, <)$ ✓
- $\text{Aut}(\text{random graph})$ ✓
- $\text{SL}(V)$ ✗

👁️ STRONG AMALGAMATION (SAP)

↳

NO ALGEBRAICITY

THEOREM (MARINON, P. '26)

$\mathcal{L}$ ~~OLIGOMORPHIC~~, NO ALGEBRAICITY

⇒ NO REGULAR MIXED IDENTITIES

👁️ $\text{Aut}(\mathbb{Q}, <)$ ~~OPEN!~~

"RICHNESS" OF $\mathcal{L}$ SHOULD AVOID
REGULAR IDENTITIES

ALGEBRAIC CLOSURE

$F \subseteq X$ FINITE

$\text{acl}(F) = \{x \in X \mid x \text{ HAS FINITE ORBIT UNDER } G_F\}$

NO ALGEBRAICITY: $\forall F \text{ } \text{acl}(F) = F$

EXAMPLE

- $\text{Sym}(X)$ ✓
- $\text{Aut}(\mathbb{Q}, <)$ ✓
- $\text{Aut}(\text{random graph})$ ✓
- $\text{GL}(V)$ ✗

👁️ STRONG AMALGAMATION (SAP)

↳

NO ALGEBRAICITY

THEOREM (MARINON, P. '26)

$\mathcal{L}$ ~~OLIGOMORPHIC~~, NO ALGEBRAICITY

⇒ NO REGULAR MIXED IDENTITIES

👁️ $\text{Aut}(\mathbb{Q}, <)$ ~~OPEN!~~

WHAT ABOUT VECTOR SPACES?

TRUE SCOPE OF THE THEOREM

TRUE SCOPE OF THE THEOREM

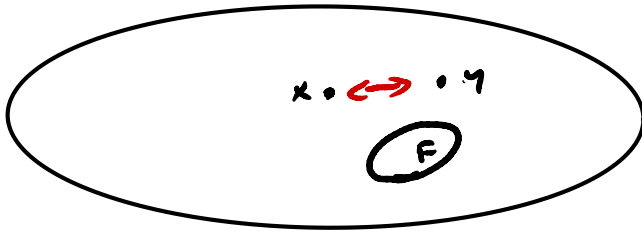
- acl SATISFIES EXCHANGE:

$$\forall x \forall y \forall F$$

$$x \in \text{acl}(F \cup \{y\}) \setminus \text{acl}(F)$$

$$\rightarrow y \in \text{acl}(F \cup \{x\})$$

"PREGEOMETRY"



TRUE SCOPE OF THE THEOREM

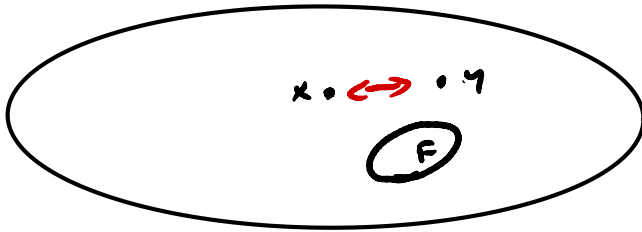
- acl SATISFIES EXCHANGE:

$$\forall x \forall y \forall F$$

$$x \in \text{acl}(F \cup \{y\}) \setminus \text{acl}(F)$$

$$\rightarrow y \in \text{acl}(F \cup \{x\})$$

"PREGEOMETRY"



- acl MODULAR:

$$\dim(A \cup B) = \dim(A) + \dim(B) - \dim(A \cap B)$$

TRUE SCOPE OF THE THEOREM

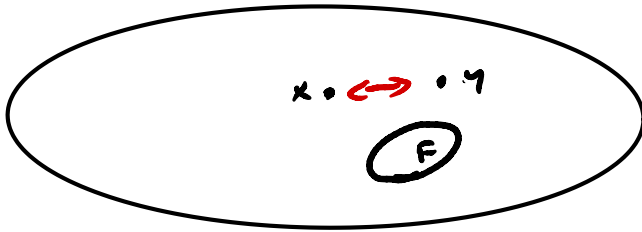
- acl SATISFIES EXCHANGE:

$$\forall x \forall y \forall F$$

$$x \in \text{acl}(F \cup \{y\}) \setminus \text{acl}(F)$$

$$\rightarrow y \in \text{acl}(F \cup \{x\})$$

"PREGEOMETRY"



- acl MODULAR:

$$\dim(A \cup B) = \dim(A) + \dim(B) - \dim(A \cap B)$$

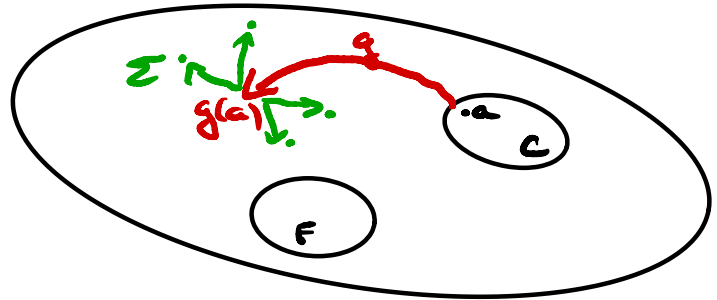
- acl HIGHER NEUMANN:

$$\forall F, C \quad \text{FINITE}$$

$$\forall a \notin \text{acl}(F)$$

$$\forall \Sigma \in G \quad \text{FINITE}$$

$$\exists g \in G_F \quad \text{acl}(\Sigma \cdot g(a)) \cap \text{acl}(C) = \text{acl}(\emptyset)$$



THEOREM (MARIMON, P. '26)

$G \cong X$, $G \in$: EXCHANGE, INFINITE DIMENSION
MODULAR
HIGHER NEUMANN

$W(x_1, \dots, x_n, y_1, \dots, y_2) = 1$ REGULAR MIXED EQUATION

$\Rightarrow$ SOLUTION SET $\subseteq G^n$ NOWHERE DENSE (POINTWISE)

THEOREM (MARIMON, P. '26)

$G \cong X$, $G \in \mathcal{A}$: EXCHANGE, INFINITE DIMENSION
MODULAR

HISHER NEUMANN

$W(x_1, \dots, x_n, y_1, \dots, y_2) = 1$ REGULAR MIXED EQUATION

$\Rightarrow$ SOLUTION SET $\subseteq G^n$ NOWHERE DENSE (POINTWISE)

POINTWISE CONVERGENCE ON G : $y \in G, S \subseteq G \Rightarrow$

$y \in \overline{S} \Leftrightarrow \forall F \in X$ FINITE

$\exists \delta \in S \quad \delta|_F = y|_F$

THEOREM (MARIMON, P. '26)

$G \cong X$, $G \in \mathcal{A}$: EXCHANGE, INFINITE DIMENSION
MODULAR
HIGHER NEUMANN

$W(x_1, \dots, x_n, y_1, \dots, y_2) = 1$ REGULAR MIXED EQUATION

$\Rightarrow$ SOLUTION SET $\subseteq G^n$ NOWHERE DENSE (POINTWISE)

POINTWISE CONVERGENCE ON G : $y \in G, S \subseteq G \Rightarrow$

$y \in \overline{S} \Leftrightarrow \forall F \in \mathcal{X} \text{ FINITE}$
 $\exists \delta \in S \quad \delta|_F = y|_F$

WHAT ABOUT PROJECTIVE LINEAR GROUPS?

THEOREM (MARIMON, P. '26)

$G \cong X$, acl : EXCHANGE, INFINITE DIMENSION
MODULAR
HIGHER NEUMANN

$W(x_1, \dots, x_n, y_1, \dots, y_2) = 1$ REGULAR MIXED EQUATION
 $\Rightarrow$ SOLUTION SET $\subseteq G^n$ NOWHERE DENSE (POINTWISE)

POINTWISE CONVERGENCE ON G : $y \in G, S \subseteq G \Rightarrow$

$y \in \overline{S} \Leftrightarrow \forall F \in X \text{ FINITE}$
 $\exists \delta \in S \quad \delta|_F = y|_F$

WHAT ABOUT PROJECTIVE LINEAR GROUPS?

acl NOT HIGHER NEUMANN: REPLACE acl BY PROJECTIVE
CLOSURE

APPLICATIONS

APPLICATIONS

- AUTOMORPHISM GROUPS OF HOMOGENEOUS STRUCTURES
WITH STRONG AMALGAMATION:

$(\mathbb{Q}, <)$, $(\mathbb{N}, =)$, RANDOM GRAPH, POSET, DIGRAPH,
URYSOHN SPACE

APPLICATIONS

- AUTOMORPHISM GROUPS OF HOMOGENEOUS STRUCTURES
WITH STRONG AMALGAMATION:

$(\mathbb{Q}, <)$, $(\mathbb{N}, =)$, RANDOM GRAPH, POSET, DIGRAPH,
ULYSSEAN SPACE

- HOMEOMORPHISM GROUPS OF:
 $\{0,1\}^{\mathbb{N}}$, $[0,1]^{\mathbb{N}}$, $\mathbb{R}^{\mathbb{N}}$, $S^{\mathbb{N}}$, ...

APPLICATIONS

- AUTOMORPHISM GROUPS OF HOMOGENEOUS STRUCTURES
WITH STRONG AMALGAMATION:

$(\mathbb{Q}, <)$, $(\mathbb{N}, =)$, RANDOM GRAPH, POSET, DIGRAPH,
ULYSSEAN SPACE

- HOMEOMORPHISM GROUPS OF:

$\{0,1\}^{\mathbb{N}}$, $[0,1]^{\mathbb{N}}$, $\mathbb{R}^{\mathbb{N}}$, $S^{\mathbb{N}}$, ...

- GENERAL LINEAR / PROJECTIVE GROUPS

;

PROOF IDEA:

PROOF IDEA:

$$w(x_1, \dots, x_n, y_1, \dots, y_r) = 1 \quad \text{REGULAR, } G \ni x$$

PROOF IDEA:

$$w(x_1, \dots, x_n, y_1, \dots, y_r) = 1 \text{ REGULAR, } G \cap X$$

$$\text{WANT: } \delta_1, \dots, \delta_n \in G, a_0 \in X$$

$$\text{SUCH THAT } w(\delta_1, \dots, \delta_n, y_1, \dots, y_r)(a_0) \neq a_0$$

PROOF IDEA:

$w(x_1, \dots, x_n, y_1, \dots, y_r) = 1$ REGULAR, $G \supset X$

WANT: $\delta_1, \dots, \delta_n \in G$, $a_0 \in X$

SUCH THAT $w(\delta_1, \dots, \delta_n, y_1, \dots, y_r)(a_0) \neq a_0$

- WLOG: $n = 1$ (SO WE JUST WANT δ, a_0)

PROOF IDEA:

$$w(x_1, \dots, x_n, y_1, \dots, y_r) = 1 \text{ REGULAR, } G \ni x$$

$$\text{WANT: } \delta_1, \dots, \delta_n \in G, a_0 \in X$$

$$\text{SUCH THAT } w(\delta_1, \dots, \delta_n, y_1, \dots, y_r)(a_0) \neq a_0$$

- WLOG: $n=1$ (so we just want δ, a_0)

- WLOG: $w = y_{10} \times y_{10}^{-1} \times y_9 \times y_9^{-1} \times y_8 \times y_8^{-1} \times y_7 \times y_7^{-1} \times y_6 \times y_6^{-1} \times y_5 \times y_5^{-1} \times y_4 \times y_4^{-1} \times y_3 \times y_3^{-1} \times y_2 \times y_2^{-1} \times y_1 \times y_1^{-1} \times y_0$

PROOF IDEA:

$$w(x_1, \dots, x_n, y_1, \dots, y_r) = 1 \text{ REGULAR, } G \cap X$$

$$\text{WANT: } \delta_1, \dots, \delta_n \in G, a_0 \in X$$

$$\text{SUCH THAT } w(\delta_1, \dots, \delta_n, y_1, \dots, y_r)(a_0) \neq a_0$$

- WLOG: $n=1$ (SO WE JUST WANT δ, a_0)
- WLOG: $w = y_{11} \times y_{10} \times^{-1} y_9 \times^{-1} y_8 \times y_7 \times y_6 \times y_5 \times^{-1} y_4 \times^{-1} y_3 \times y_2 \times y_1 \times y_0$
- PICK a_0 FREE FROM INITIAL CONDITION ON δ (FINITE)

PROOF IDEA:

$$w(x_1, \dots, x_n, y_1, \dots, y_r) = 1 \text{ REGULAR, } G \cap X$$

$$\text{WANT: } \delta_1, \dots, \delta_n \in G, a_0 \in X$$

$$\text{SUCH THAT } w(\delta_1, \dots, \delta_n, y_1, \dots, y_r)(a_0) \neq a_0$$

- WLOG: $n=1$ (SO WE JUST WANT δ, a_0)
- WLOG: $w = y_1 \times y_2 \times^{-1} y_3 \times^{-1} y_4 \times y_5 \times y_6 \times y_7 \times^{-1} y_8 \times^{-1} y_9 \times y_{10} \times y_{11} \times y_{12}$
- PICK a_0 FREE FROM INITIAL CONDITION ON δ (FINITE)
- DEFINE δ PARTIALLY SO IT SENDS $y_1 a_0$ TO NIRVANA

PROOF IDEA:

$$w(x_1, \dots, x_n, y_1, \dots, y_r) = 1 \text{ REGULAR, } G \cap X$$

$$\text{WANT: } \delta_1, \dots, \delta_n \in G, a_0 \in X$$

$$\text{SUCH THAT } w(\delta_1, \dots, \delta_n, y_1, \dots, y_r)(a_0) \neq a_0$$

- WLOG: $n=1$ (SO WE JUST WANT δ, a_0)
- WLOG: $w = y_1 \times y_0 \times^{-1} y_9 \times^{-1} y_8 \times y_7 \times y_6 \times y_5 \times^{-1} y_4 \times^{-1} y_3 \times y_2 \times y_1 \times y_0$
- PICK a_0 FREE FROM INITIAL CONDITION ON δ (FINITE)
- DEFINE δ PARTIALLY SO IT SENDS $y_0 a_0$ TO NIRVANA
- DEFINE δ FURTHER SO IT SENDS $y_1 \delta y_0 a_0$ TO NIRVANA
-

PROOF IDEA:

$w(x_1, \dots, x_n, y_1, \dots, y_r) = 1$ REGULAR, $G \cap X$

WANT: $\delta_1, \dots, \delta_n \in G$, $a_0 \in X$

SUCH THAT $w(\delta_1, \dots, \delta_n, y_1, \dots, y_r)(a_0) \neq a_0$

- WLOG: $n=1$ (SO WE JUST WANT δ, a_0)
- WLOG: $w = y_1 \times y_0 \times^{-1} y_4 \times^{-1} y_2 \times y_7 \times y_6 \times y_5 \times^{-1} y_4 \times^{-1} y_3 \times y_2 \times y_1 \times y_0$
- PICK a_0 FREE FROM INITIAL CONDITION ON δ (FINITE)
- DEFINE δ PARTIALLY SO IT SENDS $y_0 a_0$ TO NIRVANA
- DEFINE δ FURTHER SO IT SENDS $y_1 \delta y_0 a_0$ TO NIRVANA
- $\vdots$
- BUT $\delta^{-1} y_3 \delta y_2 \delta y_1 \delta y_0(a_0)$ MIGHT BE EQUAL TO $y_2 \delta y_1 \delta y_0(a_0)$!

PROOF IDEA:

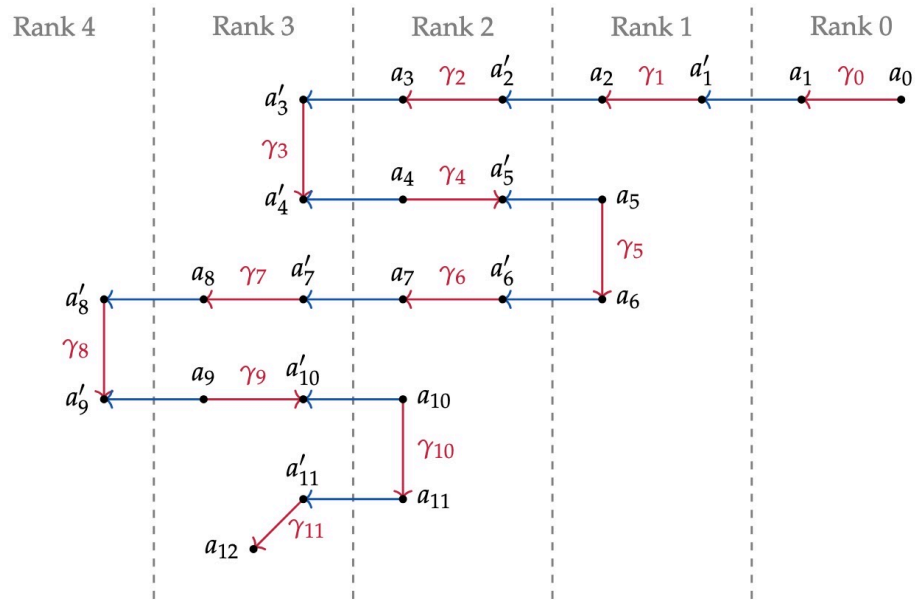
$$w(x_1, \dots, x_n, y_1, \dots, y_r) = 1 \text{ REGULAR, } G \cap X$$

$$\text{WANT: } \delta_1, \dots, \delta_n \in G, a_0 \in X$$

$$\text{SUCH THAT } w(\delta_1, \dots, \delta_n, y_1, \dots, y_r)(a_0) \neq a_0$$

- WLOG: $n=1$ (SO WE JUST WANT δ, a_0)
- WLOG: $w = y_1 \times y_2 \times \dots \times y_n$
- PICK a_0 FREE FROM INITIAL CONDITION ON δ (FINITE)
- DEFINE δ PARTIALLY SO IT SENDS $y_1 a_0$ TO NIRVANA
- DEFINE δ FURTHER SO IT SENDS $y_1 \delta y_2 a_0$ TO NIRVANA
- $\vdots$
- BUT $\delta^{-1} y_3 \delta y_2 \delta y_1 \delta y_0(a_0)$ MIGHT BE EQUAL TO $y_2 \delta y_1 \delta y_0(a_0)$!

HAVE TO THINK AHEAD



$$\gamma_{11} \times \gamma_{10} \times^{-1} \gamma_9 \times^{-1} \gamma_8 \times \gamma_7 \times \gamma_6 \times \gamma_5 \times^{-1} \gamma_4 \times^{-1} \gamma_3 \times \gamma_2 \times \gamma_1 \times \gamma_0 = 1$$

THEOREM (MARIMON, P. '26)

$\mathcal{G} \ni X$, $\mathcal{G} \ni$: EXCHANGE, INFINITE DIMENSION
MODULAR
HIGHER NEUMANN

$W(x_1, \dots, x_n, y_1, \dots, y_r) = 1$ REGULAR MIXED EQUATION

$\Rightarrow$ SOLUTION SET $\subseteq \mathcal{G}^n$ NOWHERE DENSE (POINTWISE)

Thank you!

FUNDING STATEMENT: FUNDED BY THE EUROPEAN UNION (ERC, PROJECT, 101074674).
VIEWS AND OPINIONS EXPRESSED ARE HOWEVER THOSE OF THE AUTHOR(S) ONLY
AND DO NOT NECESSARILY REFLECT THOSE OF THE EUROPEAN UNION
OR THE EUROPEAN RESEARCH COUNCIL GRANT AGENCY.
NEITHER THE EUROPEAN UNION NOR THE GRANTING AUTHORITY
CAN BE HELD RESPONSIBLE FOR THEM.