BINARY SYMMETRIES OF TRACTABLE NON-RIGID STRUCTURES

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TU WIEN

THE CONSTRAINT SATISFACTION PROBLEM: COMPLEXITY & APPROXIMABILITY

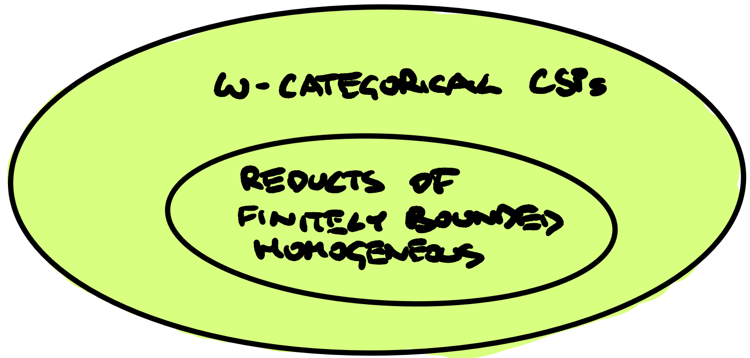
DAGSTUHL 05/2025

EUROPEAN RESEARCH COUNCIL

ERC SYNERGY GRANT POCOCOP (GA 101071674)

FWF I5948

MO!


ω-CATEGORICAL CSPs
REDUCTS OF
FINITELY BOUNDED
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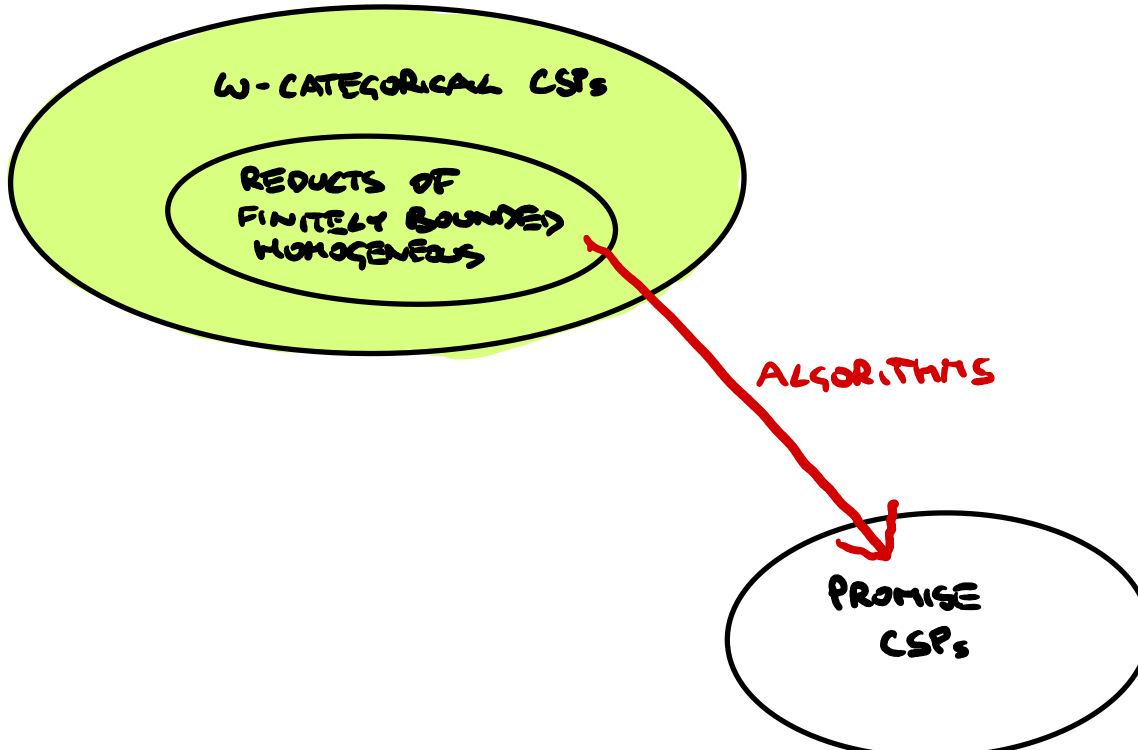
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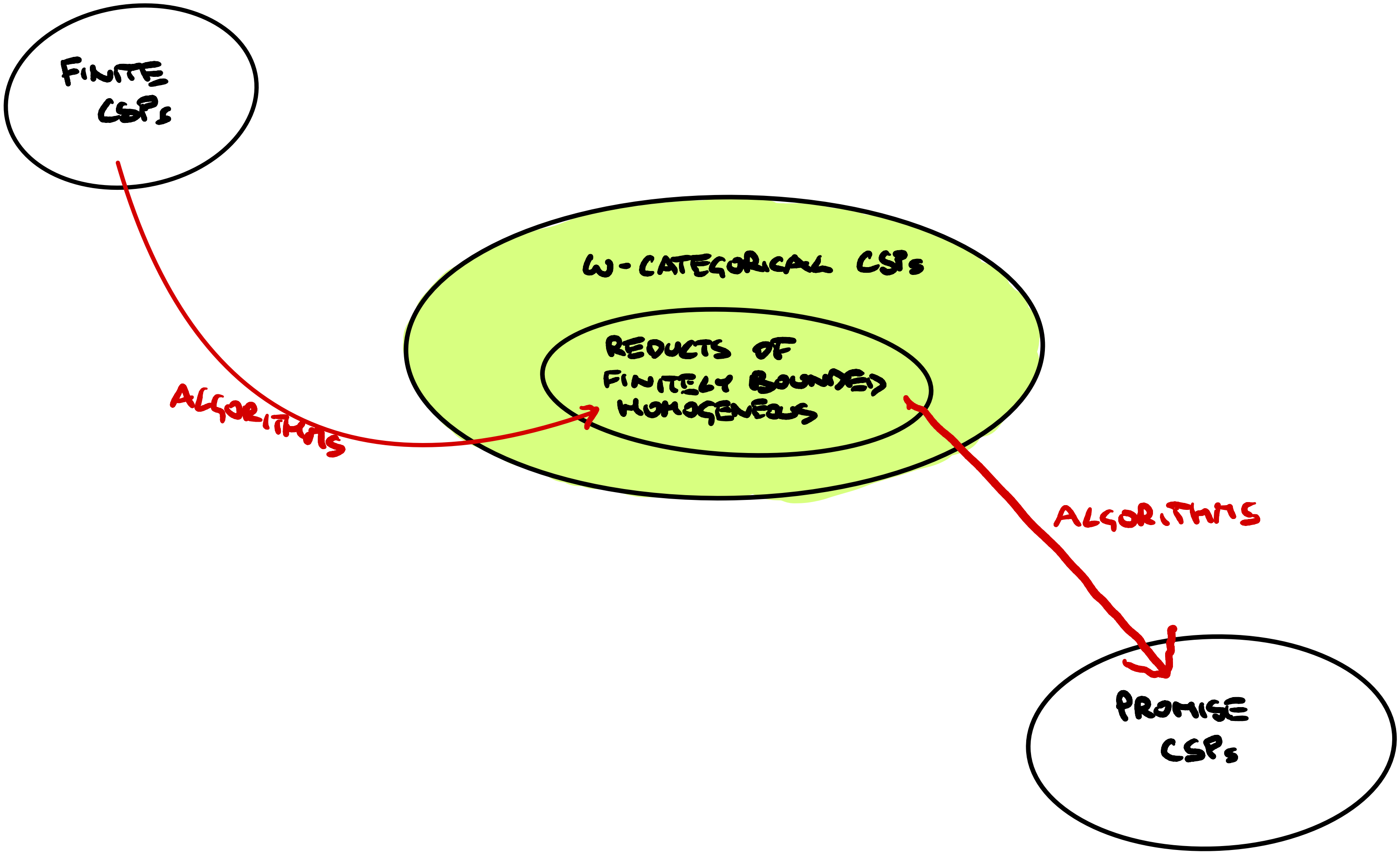
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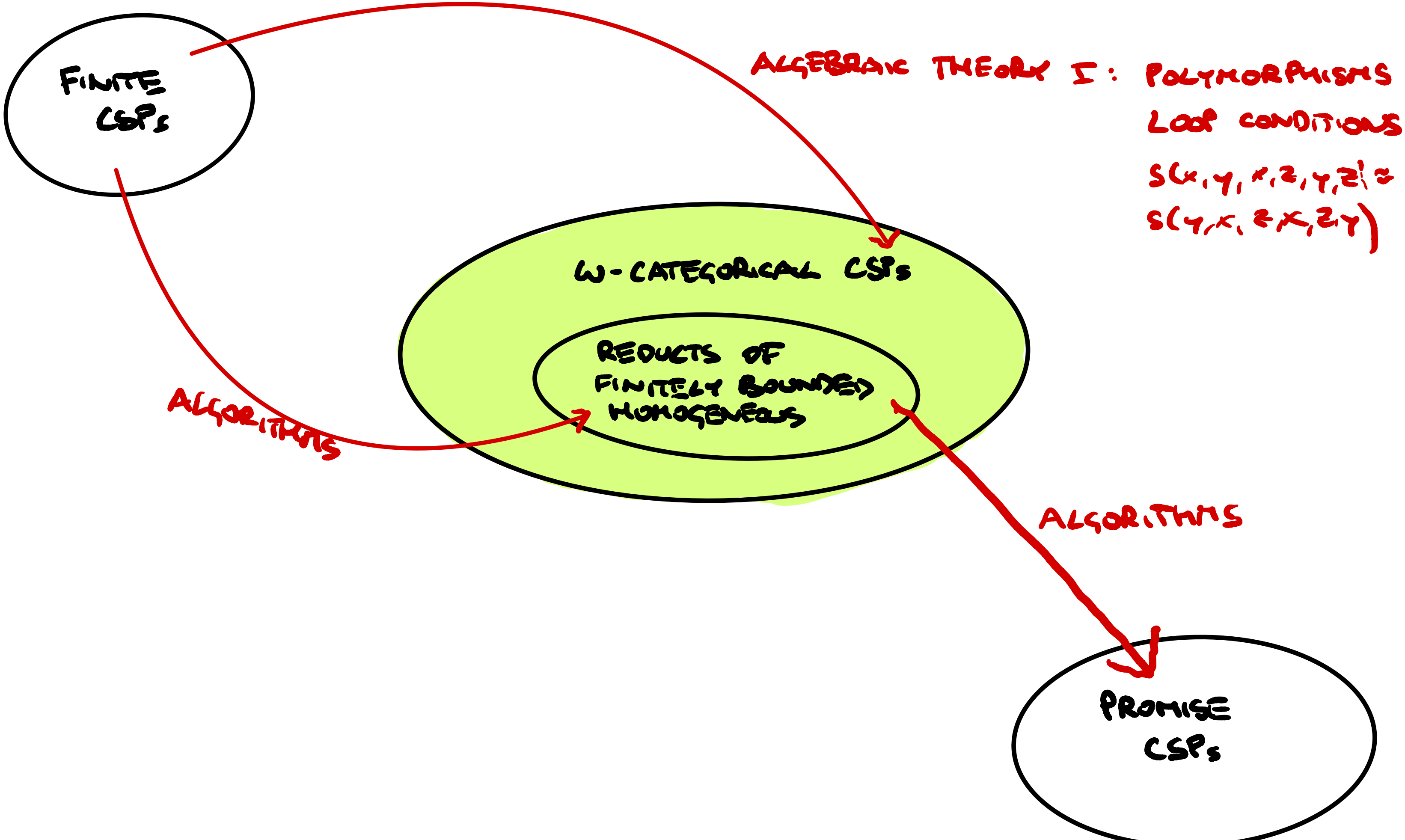
FINITE CSPs
ALGEBRAIC THEORY I: POLYMORPHISMS
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s(x,y,x,z,y,z) ≈ s(y,x,z,x,z,y)
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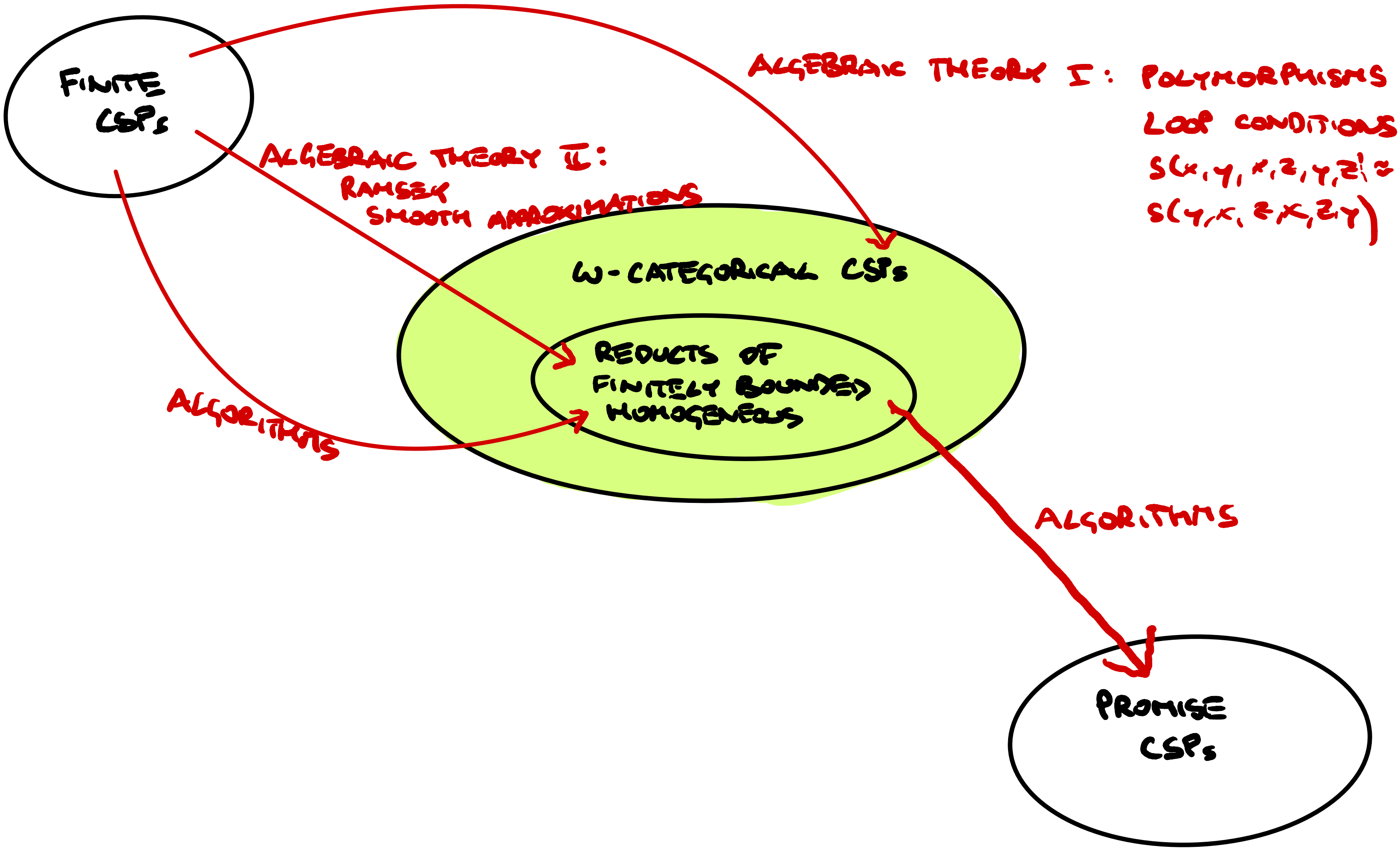
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$$\overline{\mathrm{Aut}\,\mathbb{A}} = \mathrm{End}(\mathbb{A})$$

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WHEN DO WE KNOW CSP($\mathbb{A}$) IS NP-HARD?

CERTAINLY IF Pol($\mathbb{A}$) ESSENTIALLY UNARY:

$$\forall f(x_1, \dots x_n) \; \exists i \; \exists g \quad f(x_1, \dots x_n) \approx g(x_i)$$

$f \mapsto$ PROJ$_i$ CONTINUOUS CLONE HOMOMORPHISM

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MINIMAL AMOUNT OF STRUCTURE IF $\mathrm{Pol}(\mathbb{A})$ ESSENTIAL?

(NOT ESS. UNARY)

THM (ROSENBERG '80)

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OR

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OR $m(x,x,y) = m(x,y,x) = m(y,x,x) = x$

- $x+y+z$ FOR + A BOOLEAN GROUP ACTION

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- A SEMIPROJECTION f

$\exists i \quad f(x_1 \dots x_n) = x_i$

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FUN FACT: USE RAMSEY'S THM

PROVING TRACTABILITY IN PRACTICE:

FIND BINARY ESS. POLYMORPHISM, BUILD ON THIS.

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Lemma 40. *Let $f\colon V^k \to V$ be an essential operation. Then f generates a binary essential operation.*

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Lemma 40. *Let $f\colon V^k \to V$ be an essential operation. Then f generates a binary essential operation.*

LEMMA 6.1.29. *Let $\mathscr{C}$ be a clone with an essential operation that contains a permutation group $\mathscr{G}$ with the orbital extension property. Then $\mathscr{C}$ must also contain a binary essential operation.*

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COROLLARY 6.1.31. *Let $\mathfrak{B}$ be an infinite 2-set-transitive structure with an essential polymorphism. Then $\mathfrak{B}$ also has a binary essential polymorphism.*

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Proposition 22. *Let $\mathbb{A}$ be a first-order reduct of a homogeneous structure $\mathbb{B}$ such that $\mathbb{B}$ has a free orbit. If $\mathrm{Pol}(\mathbb{A})$ contains an essential function, then it contains a binary essential operation.*

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IN PARTICULAR: BINARY INJECTION

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Theorem 7.2.1. *Let $\mathfrak{B}$ be a structure with a 3-transitive automorphism group such that* $\mathrm{Pol}(\mathfrak{B})$ *contains an essential operation but no constant operation. Then* $\mathrm{Pol}(\mathfrak{B})$ *also contains a binary injective operation.*

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LEMMA 3.6. *Let $f\colon H_n^2 \to H_n$ be a binary essential function that preserves E and N. Then f generates a binary injection.*

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(2) Does every primitive oligomorphic permutation group have the orbital extension property (Definition 6.1.28)? $\Rightarrow$ BINARY ESSENTIAL

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THM [BODIRSKY + CHEN '07] MARIMON + P. '25

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THM (P. + RYDVAL + SCHÖBI + SPIESS '25)

$\mathbb{A}$ DATALOG-INTERREDUCIBLE WITH POL-INJECTIVE $\mathbb{A}'$

$\mathrm{Pol}(\mathbb{A})$ EQ. NON-TRIVIAL $\Leftrightarrow$ $\mathrm{Pol}(\mathbb{A}')$ EQ. NON-TRIVIAL

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(∗) $\not\Longleftarrow$

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EXAMPLE

$\mathbb{A}$ ... "ORDER CONSTRAINT LANGUAGE"

"CONSTRAINTS ONLY DEPEND ON ORDER OF ELEMENTS"

"$\mathbb{A}$ REDUCT OF $(\mathbb{Q}, <)$"

"TEMPORAL TEMPLATE"

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NONETHELESS: INGREDIENT FOR COMPLEXITY [BODIRSKY & KARA '08]

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THM (BRUNAR & P. & SCHÖBI DAGSTUHL '25)

$\text{Pol}(\mathbb{A})$ EQU. NON-TRIVIAL $\Rightarrow$ $\text{Pol}(\mathbb{A}) \models \alpha\, s(a,r,e,a) \approx \beta\, s(r,a,r,e)$

When you solve a CSP
you must avoid the graph K_3
the graph K_3 will follow you
unless you strike with arity two

Thank you!

THM (MARIMON + P. '25)

$\mathbb{A}$ ω-CATEGORICAL OR FINITE & $\mathrm{Aut}(\mathbb{A})$ NOT FREE BOOLEAN
$\mathrm{Pol}(\mathbb{A})$ EQU. NON-TRIVIAL $\Rightarrow$ $\bullet$ BINARY