TBA

Michael Pinsker (Paris 7)

joint work with Manuel Bodirsky (LIX Palaiseau)

Dagstuhl 2012
Topological Birkhoff & Applications

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Outline

Topological Birkhoff theorem

Generalization of Birkhoff’s HSP fin theorem from finite to certain infinite algebras

Corollary in the purely model theoretic language: Primitive positive interpretations

Applications to CSPs with infinite templates

Implication chain: ↓

Motivation chain: ↑

TBA

Michael Pinsker (Paris 7)
Outline

Topological Birkhoff
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- Generalization of Birkhoff’s $\text{HSP}^\text{fin}$ theorem from finite to certain infinite algebras
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- Generalization of Birkhoff’s HSP$^\text{fin}$ theorem from finite to certain infinite algebras
- Corollary in the purely model theoretic language: Primitive positive interpretations
- Applications to CSPs with infinite templates
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- Generalization of Birkhoff’s $\text{HSP}^\text{fin}$ theorem from finite to certain infinite algebras
- Corollary in the purely model theoretic language: Primitive positive interpretations
- Applications to CSPs with infinite templates

Implication chain: $\downarrow$ (TBA)
Motivation chain: $\uparrow$ (ATB)
Part I: Simple cloning
Let \( \Gamma \) be a relational structure with finite language \( \tau \).

\[ \text{CSP}(\Gamma) \]

**INPUT:** A finite set of variables and \( \tau \)-constraints on these variables.

**QUESTION:** Does there exists a satisfying assignment of values in \( \Gamma \)?
Let $\Gamma$ be a relational structure with finite language $\tau$.

**CSP($\Gamma$)**

**INPUT:** A finite set of variables and $\tau$-constraints on these variables.

**QUESTION:** Does there exist a satisfying assignment of values in $\Gamma$?

$\Gamma$ can be infinite!
Three basic human fears

Fear 1: the fear of nonexistence
Q: Every thing in my life is finite. Why should $\Gamma$ be infinite?
A1: You don't have to invite the elements of $\Gamma$ to your living room!
A2: Are the natural numbers part of your life?

Fear 2: the fear of impotence
Q: How can an algorithm calculate anything about infinite $\Gamma$?
A: How can an algorithm add integers?
Q: Aren't there undecidable infinite template CSPs?
A1: Isn't ... undecidable too?
A2: There is a large interesting class of infinite $\Gamma$ whose CSP is in NP.

Fear 3: the fear of meaninglessness
Q: Can CSP $(\Gamma)$ be meaningful for infinite $\Gamma$?
A: Is acyclicity of digraphs a meaningful problem?
Q: Why do you generalize?
A: Why did you restrict? OK for technical reasons.
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Finite simple cloning
Finite simple cloning ("Neanderthal cloning")

Theorem (Geiger '68; Bodnarchuk+Kaluzhnin+Kotov+Romov '69)

Let $\Gamma, \Delta$ be finite relational structures on the same domain. TFAE:

1. $\Delta$ is pp-definable in $\Gamma$.
2. $\text{Pol}(\Gamma) \subseteq \text{Pol}(\Delta)$.

$\Rightarrow$ $\Delta$ "sits inside" $\Gamma$.

$\Rightarrow$ $\text{CSP}(\Gamma)$ is at least as hard as $\text{CSP}(\Delta)$.

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\(\omega\)-categoricity

A countable relational structure \(\Gamma\) is \(\omega\)-categorical iff its theory has no countable non-standard model.

Finiteness condition!

Meaning: For every \(n \geq 1\) there exist finitely many \(n\)-tuples \(a_1, \ldots, a_k\) of elements of \(\Gamma\) such that any other \(n\)-tuple is equivalent to one of the \(a_i\) with respect to the theory of \(\Gamma\).

Examples: Order of rationals, random graph, random partial order.

Non-example: Order of integers.

CSP: essentially finitely many choices for \(n\) variables!
**ω-categoricity**

**Def.** A countable relational structure $\Gamma$ is **ω-categorical** iff its theory has no countable non-standard model.

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Theorem (Bodirsky+Nešetřil '03)

Let $\Gamma$, $\Delta$ be $\omega$-categorical rel. structures on the same domain. TFAE:

$\Delta$ is $pp$-definable in $\Gamma$;

$Pol(\Gamma) \subseteq Pol(\Delta)$.

$\Rightarrow \Delta$ "sits inside" $\Gamma$.

$\Rightarrow \text{CSP}(\Gamma)$ is at least as hard as $\text{CSP}(\Delta)$.

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Part II: Double cloning
Interpretations

Definition. Let $\Delta$, $\Gamma$ be relational structures. $\Delta$ has a pp-interpretation in $\Gamma$ iff it is constructible from $\Gamma$ by expanding $\Gamma$ by all pp-definable relations; then taking a finite “power”; then taking a substructure induced by a pp-definable subset; then factoring by a pp-definable equivalence relation; then forget some of the relations.

Meaning. $\Rightarrow \Delta$’s sits inside $\Gamma$ in a weaker sense. $\Rightarrow \text{CSP}(\Gamma)$ is at least as hard as $\text{CSP}(\Delta)$.

Example: $(\mathbb{Q}; +, \cdot)$ has a pp-interpretation in $(\mathbb{Z}; +, \cdot)$. 

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Meaning. $\Rightarrow$ $\Delta$ “sits inside” $\Gamma$ in a weaker sense.
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Can view \( \Gamma \) as an algebra on \( \Gamma \) by giving it a signature.

Let \( C \) be a class of algebras of the same signature.

\[ \text{Pol}(\Gamma) \]

... all finite products of algebras in \( C \).

\[ \text{HSP}_{\text{fin}}(\text{Pol}(\Gamma)) \]

... all subalgebras of algebras in \( C \).

\[ \text{H}(\Gamma) \]

... all factors of algebras in \( C \).

Theorem

Let \( \Gamma, \Delta \) be finite. TFAE:

- \( \Delta \) has a pp-interpretation in \( \Gamma \);
- there exists \( B \in \text{HSP}_{\text{fin}}(\text{Pol}(\Gamma)) \) whose functions are elements of \( \text{Pol}(\Delta) \).

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Can view \( \text{Pol}(\Gamma) \) as an algebra on \( \Gamma \) by giving it a signature.

Let \( \mathcal{C} \) be a class of algebras of the same signature.

- \( P^{\text{fin}}(\mathcal{C}) \) . . . all finite products of algebras in \( \mathcal{C} \).
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- \( H(\mathcal{C}) \) . . . all factors of algebras in \( \mathcal{C} \).
Finite double cloning I

Can view $\text{Pol}(\Gamma)$ as an algebra on $\Gamma$ by giving it a signature.

Let $\mathcal{C}$ be a class of algebras of the same signature.

- $\text{P}^{\text{fin}}(\mathcal{C})$ . . . all finite products of algebras in $\mathcal{C}$.
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**Theorem**

Let $\Gamma, \Delta$ be finite. TFAE:

- $\Delta$ has a pp-interpretation in $\Gamma$;
- there exists $\mathcal{B} \in \text{HSP}^{\text{fin}}(\text{Pol}(\Gamma))$ whose functions are elements of $\text{Pol}(\Delta)$.
Theorem (Birkhoff)

Let $\mathcal{A}, \mathcal{B}$ be finite $\tau$-algebras. TFAE:

- $\mathcal{B} \in \text{HSP}^{\text{fin}}(\mathcal{A})$.
- all equations of $\mathcal{A}$ also hold in $\mathcal{B}$.
- the natural homomorphism which sends every $\tau$-term in $\mathcal{A}$ to the corresponding term in $\mathcal{B}$ exists.
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Theorem

Let $\Gamma, \Delta$ be finite relational structures. TFAE:

- $\Delta$ has a pp-interpretation in $\Gamma$;
- there exists a homomorphism from $\text{Pol}(\Gamma)$ into $\text{Pol}(\Delta)$. 
Finite double cloning visualized

\[ \Gamma \rightarrow \text{Pol}(\Gamma) \]

\[ \text{Equ}(\text{Pol}(\Gamma)) \]
Let $S$ be the structure on $\{0, 1\}$ with the only relation $\{(0, 0, 1), (0, 1, 0), (1, 0, 0)\}$.

$\text{CSP}(S)$ equivalent to positive 1-in-3-SAT. NP-complete.

$\text{Pol}(S)$ is the trivial clone consisting only of projections.

Fact

Let $\Gamma$ be finite. TFAE:

- $S$ has a pp-interpretation in $\Gamma$.
- There exists a homomorphism from $\text{Pol}(\Gamma)$ onto $1$.
- All finite structures have a pp-interpretation in $\Gamma$.

Conjecture (Bulatov+Jeavons+Krokhin; Feder+Vardi)

For finite idempotent cores $\Gamma$ this is the unique reason for NP-hardness.
Dichotomy?

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- $S$ has a pp-interpretation in $\Gamma$.
- There exists a homomorphism from $\text{Pol}(\Gamma)$ onto $1$.
- All finite structures have a pp-interpretation in $\Gamma$.

Conjecture (Bulatov+Jeavons+Krokhin; Feder+Vardi)

For finite idempotent cores $\Gamma$ this is the unique reason for
NP-hardness.
Theorem
Let $\Gamma$ be $\omega$-categorical, and $\Delta$ be arbitrary. TFAE:

- $\Delta$ has a pp-interpretation in $\Gamma$;
- there exists $B \in \text{HSP}_{\text{fin}}(\text{Pol}(\Gamma))$ whose functions are elements of $\text{Pol}(\Delta)$.

What are the elements of $\text{HSP}_{\text{fin}}(\text{Pol}(\Gamma))$? Birkhoff help!

Theorem for which algebras instead of finite ones?

Def. A permutation group on $X$ is oligomorphic iff its action on $X^n$ has finitely many orbits for all $n \geq 1$.

Def. An algebra is oligomorphic iff its term functions contain an oligomorphic permutation group.

Thm. A relational structure $\Gamma$ is $\omega$-categorical iff $\text{Pol}(\Gamma)$ is oligomorphic.
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Topological Birkhoff

Birkhoff for oligomorphic algebras?

Every clone on an infinite domain carries two kinds of structure:

an algebraic structure: composition (aka equations);

topological structure:

a sequence $(g_n)_{n \in \omega}$ of $m$-ary functions converges to an $m$-ary function $f$ iff for all finite subsets $A$ of the domain there is $j \in \omega$ such that $g_i$ agrees with $f$ on $A$ for all $i \geq j$.

Theorem ("Topological Birkhoff" MB+MP '12)

Let $A, B$ be oligomorphic $\tau$-algebras. TFAE:

$B \in \text{HSP}_{\text{fin}}(A)$.

the natural homomorphism which sends every $\tau$-term in $A$ to the corresponding term in $B$ exists and is continuous.
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Theorem (MB+MP '12)

Let $\Gamma, \Delta$ be $\omega$-categorical or finite relational structures. TFAE:

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Let $\Gamma$ be $\omega$-categorical. TFAE:

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3. All finite structures have a pp-interpretation in $\Gamma$.

Example: $\Gamma := (\mathbb{Q}; \{ (x, y, z) \in \mathbb{Q}^3 | x < y < z \lor z < y < x \})$

$\text{CSP}(\Gamma)$ is called the Betweenness problem.

Let $f \in \text{Pol}(\Gamma)$ of arity $k$. There is a unique $i \in \{1, \ldots, k\}$ such that:

1. $\forall x, y \in \Gamma^k: (\forall j x_j \neq y_j \land x_i < y_i) \Rightarrow f(x) < f(y)$, or
2. $\forall x, y \in \Gamma^k: (\forall j x_j \neq y_j \land x_i < y_i) \Rightarrow f(x) > f(y)$.

Set $\xi(f)$ to be the $i$-th $k$-ary projection in $\mathbb{1}$.

Straightforward: $\xi: \text{Pol}(\Gamma) \to \mathbb{1}$ is a continuous homomorphism.
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Part III: Infinite triple cloning
Automatic continuity

Does the complexity of CSP (Γ) only depend on the algebraic structure of Pol (Γ)?

Automatic continuity for automorphism groups: there is a model of ZF (+DC) where every homomorphism between automorphism groups is continuous (Shelah'84).

TBA

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Do there exist $\omega$-categorical $\Gamma, \Delta$ such that $\text{Pol}(\Gamma), \text{Pol}(\Delta)$ are isomorphic algebraically but not topologically?

Yes for automorphism groups (Evans+Hewitt'90).

If so, when does the algebraic structure of $\text{Pol}(\Gamma)$ determine the topological one?

For automorphism groups: “small index property”.

$(N;=) (Dixon+Neumann+Thomas'86) (Q;<) (Truss'89)$

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Transactions of the AMS / arXiv.
Thank you!