- $(P, \leq)$ is a partial order (p.o.) (it might be more exact to use the term quasi-order) if $\leq$ is transitive and reflexive. (Antisymmetry is not rewuired)
- We only consider p.o.s that contain a largest element 1.
- Elements of P will be called "conditions". Larger elements are "weaker" conditions; smaller ones are "stronger". So 1 is the weakest condition.
- $D \subseteq P$ is dense, if for all $p \in P$ there is a $q \leq p$ such that $q \in D$.
- $O \subseteq D$ is open, if for all $p \in O$ and $q \leq p$ also $q \in O$.
- D is open dense if it is open and dense.
- For every $X \subseteq P$ there is a minimal open set O containing X:

  $$O = \{q \in P : (\exists p \in X)\, q \leq p\}.$$

  We call O "downwards closure" of X.
- p is compatible with q, or: $p \parallel q$, if there is an $r \leq p, q$ in P. Otherwise p and q are incompatible, or: $p \perp q$.
- D is predense, if for all $p \in P$ there is a $p' \in D$ such that $p \parallel p'$
- Exercise: D is predense iff the downwards closure of D is dense.
- $A \subseteq P$ is an antichain, if all $p \neq q$ in A are incompatible.
- $A \subseteq P$ is a maximal antichain, if A is an antichain, and no proper superset of A is an antichain.
- Exercise: Show: A is maximal antichain iff it is a predense antichain.
- $G \subseteq P$ is a filter, if $q \in G$ and $p \geq q$ implies $p \in G$ and if for all p, q in G there is an $r \leq p, q$ in G (note that this is stronger than just: $p \parallel q$, since the witness has to be in G.)
- Assume M is a countable transitive model of ZFC, and that P is a p.o. in M. We call $G \subseteq P$ (which is generally NOT in M) M-generic, if G is a filter and if for all dense subsets $D \in M$ we get $G \cap D \neq \emptyset$.
- Exercise: Show that the definition of generic is equivalent if we replace "dense" by any of the following: "open dense", "predense", "maximal antichain". (For this, we assume that M satisfies AC.)
- Exercise: Show that G is M-generic is equivalent to: $q \in G$ and $p \geq q$ implies $p \in G$ and p, q in G implies $p \parallel q$ and for all dense subsets $D \in M$ we get $G \cap D \neq \emptyset$.
- Exercise: If M is a *countable* transitive model of ZFC, and $P \in M$, then there is an M-generic filter G (not necessarily in M).

  Hint: There are only countably many dense sets in M, enumerate then as $D_0, D_1, \ldots$. Choose $p_0 \in D_0$, and $p_{n+1} \leq p_n$ in D_n. Generate a filter from $\{p_0, p_1, \ldots\}$.
- P is seperative, if for all $q \not\leq p$ there is a $r \leq q$ such that $r \perp q$.
- Exercise: If P is seperative, then there is no M-generic filter G that is in M.
- Exercise: Show that $2^{<\omega}$, the set of all finite 0-1-sequences ordered by extension, is a seperative p.o. (it is even a tree, in particular p and q are compatible iff they are comparable, i.e., $p \leq q$ or $q \leq p$).
- Exercise: Show that the family of infinite subsets of ω, ordered by $A \leq B$ iff $A \setminus B$ is finite, is a seperative p.o.