VU Diskrete Mathematik

Exercises for Nov 10 – Session 1, 2023

25) Let G_n denote the n-dimensional hypercube. Show that G_n is Hamiltonian if $n \geq 2$.

26) Prove that every simple, connected and planar graph with at least 3 vertices satisfies $\alpha_1(G) \leq 3\alpha_0(G) - 6$. Show that this implies that K_5 is not planar.

27) Let $G = (V, E)$ be a simple, connected graph where each vertex has degree 3. Furthermore, assume that G is planar and that every vertex lies on the boundary of exactly three faces, one having a boundary consisting of 6 edges, the other two one of 4 edges each. Determine $\alpha_0(G)$, $\alpha_1(G)$, $\alpha_2(G)$ and draw a plane graph which is isomorphic to G.

28) Follow the hint below to construct a schedule for the matches in a league of $2n$ teams which meets the following constraints:

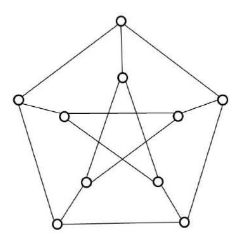
(a) In each round each team plays exactly one match.

(b) In the end each team must have played against each of the other teams exactly once.

Hint: Consider the graph K_{2n} on the vertex set $\{1, 2, \ldots, 2n\}$ and show that each of the sets $M_i = \{1i\} \cup \{xy \mid x + y \equiv 2i \mod 2n-1 \text{ and } x \neq y,\ x \neq 1,\ y \neq 1\}$ is a perfect matching (for $i = 2, \ldots, 2n$).

29) Show that the n-dimensional hypercube ($n > 1$) has a perfect matching.

30) Determine the chromatic number of the following graph.



Likewise, determine the chromatic number of the line graph of the graph above.

31) Show that a graph is bipartite if and only if there is a feasible coloring with two colors.

32) Show that a graph which has exactly one odd cycle has chromatic number 3.