VU Diskrete Mathematik

Exercises for Oct 27, 2023

17) Let (E, S_1) and (E, S_2) be matroids. Examine whether $(E, S_1 \cap S_2)$ is a matroid. Give a proof or a concrete counter-example!

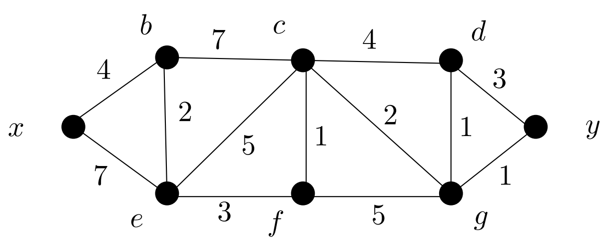
18) Let $M = (E, S)$ be a matroid and $\mathcal{B}$ the family of all its bases. Let $A, B \in \mathcal{B}$ such that $A \neq B$. Prove that

(a) neither of the inclusions $A \subseteq B$ and $B \subseteq A$ holds,

(b) for each $x \in A$ there exists $y \in B$ such that $(A \setminus \{x\}) \cup \{y\} \in \mathcal{B}$.

Hint: Show first that, if $x \in A \cap B$ in (b), then by (a) we must have $y = x$, and if $x \in A \setminus B$, then we must have $y \in B \setminus A$.

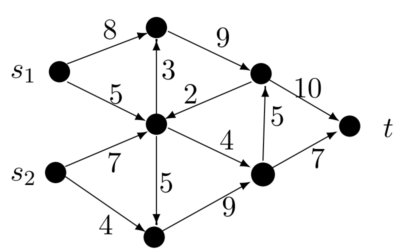
19) Use Dijkstra's algorithm to determine $d(x, y)$ in the following graph.



20) Find a graph $G = (V, E)$ and two vertices $x, y \in V$ such that Dijkstra's algorithm does not compute the distance $d(x, y)$ correctly.

21) Let ϕ_1 and ϕ_2 be to flows on a flow network $G = (V, E, w, s, t)$. Moreover, let γ_1, γ_2 be to nonnegative reals satisfying $\gamma_1 + \gamma_2 \leq 1$. Prove that $\gamma_1\phi_1 + \gamma_2\phi_2$ is a flow on G.

22) Use the algorithm of Ford and Fulkerson to compute a maximal flow in the network below!



23) For a simple and undirected graph G we define the *line graph* $\bar{G}$ as follows: $V(\bar{G}) = E(G)$ and $(e, f) \in E(\bar{G})$ if and only if the edges e and f share a vertex. Let G be a simple, undirected and connected graph for which every vertex has degree r. Prove that the line graph $\bar{G}$ of G is Eulerian.

24) For which m and n is the complete bipartite graph $K_{m,n}$ an Eulerian graph?