EDGE-COLORING PROBLEMS WITH FORBIDDEN PATTERNS AND PLANTED COLORS

JOINT WORK WITH

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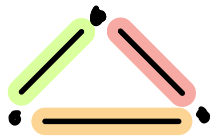
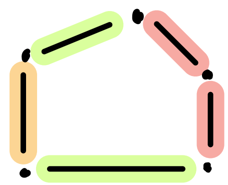
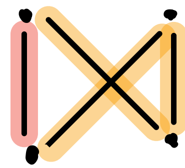
DAVIDE PERINTI

INTRODUCTION

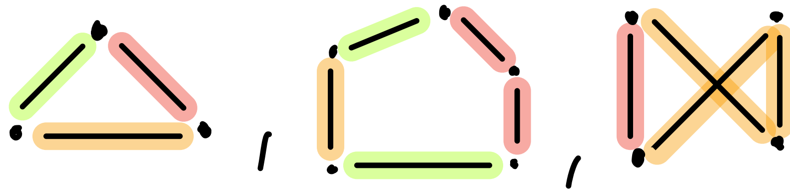
INTRODUCTION - EDGE COLORING PROBLEM

FIX A SET OF COLORS = {  }

FIX $\mathcal{F}$ = {  ,  ,  , ... }

INTRODUCTION - EDGE COLORING PROBLEM

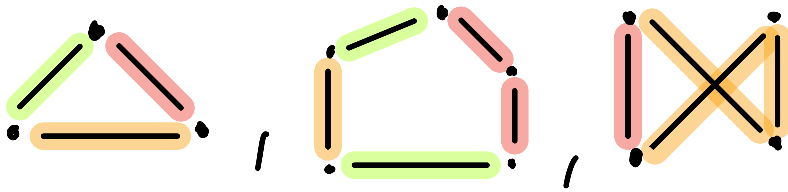
FIX A SET OF COLORS = {  }

FIX $\mathcal{F}$ = {  , ... }

INPUT: A GRAPH G

INTRODUCTION - EDGE COLORING PROBLEM

FIX A SET OF COLORS = $\{ \quad \}$

FIX $\mathcal{F} = \{$  $, \dots \}$

INPUT: A GRAPH G

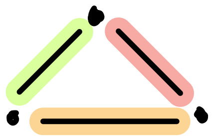
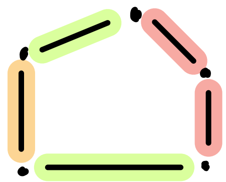
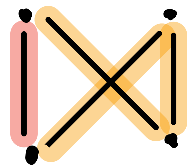
TASK: $\exists \xi : E(G) \longrightarrow$ COLORS SUCH THAT

$$\forall (F,\chi) \in \mathcal{F} \qquad (F,\chi) \not\longrightarrow (G,\xi) \; ?$$

HOMOMORPHISM

INTRODUCTION - EDGE COLORING PROBLEM

FIX A SET OF COLORS $=$ {  }

FIX $\mathcal{F} =$ {  ,  ,  , ... }

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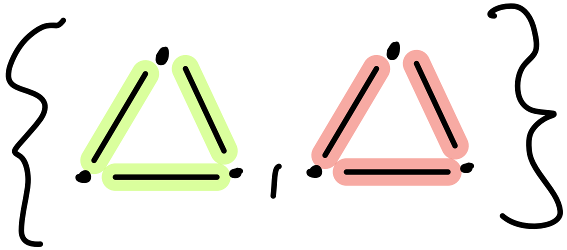
HOMOMORPHISM

WE INDICATE SUCH PROBLEM $\mathrm{Col}(\mathcal{F})$

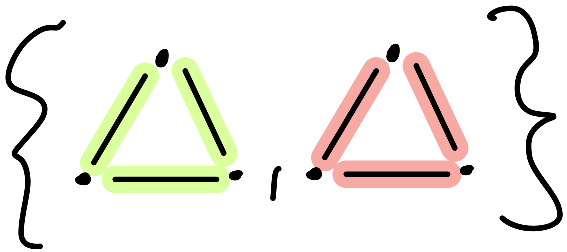
INTRODUCTION - EDGE COLORING PROBLEM

EXAMPLE

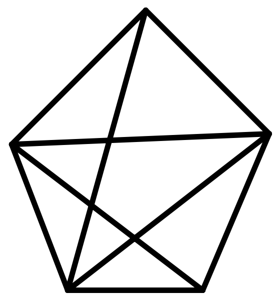
$F =$ 

INTRODUCTION - EDGE COLORING PROBLEM

EXAMPLE

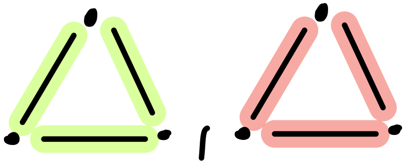
$F :=$ 

INPUT:

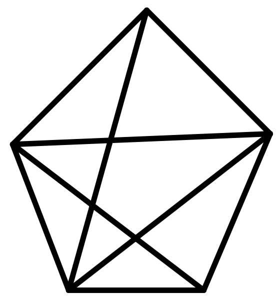
$G_1 :=$  ?

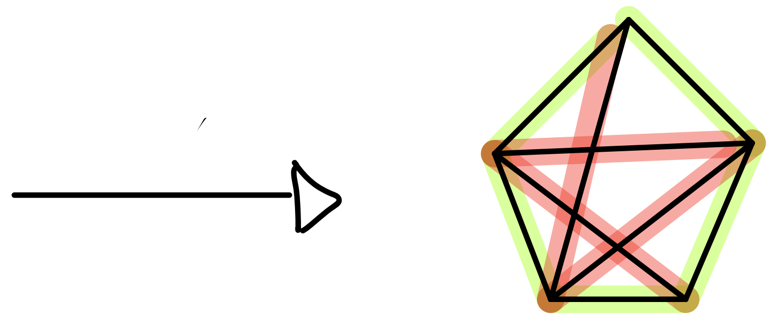
INTRODUCTION - EDGE COLORING PROBLEM

EXAMPLE

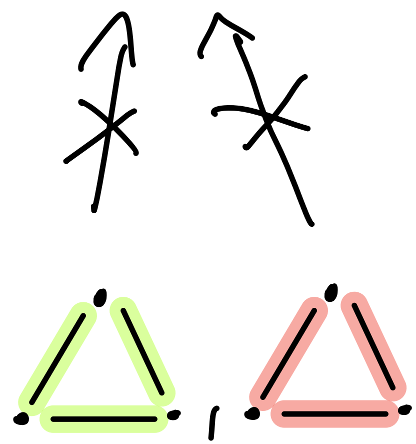
$F = \{$  $\}$

INPUT:

$G_1 :=$ 

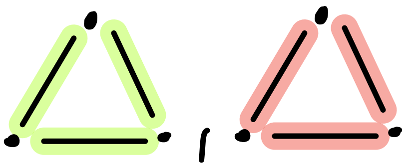


YES



INTRODUCTION - EDGE COLORING PROBLEM

EXAMPLE

$F := \{$  $\}$

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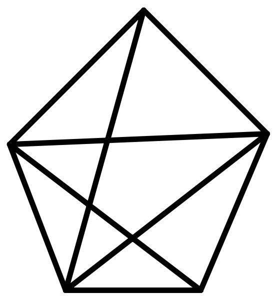
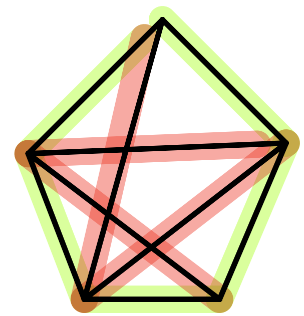
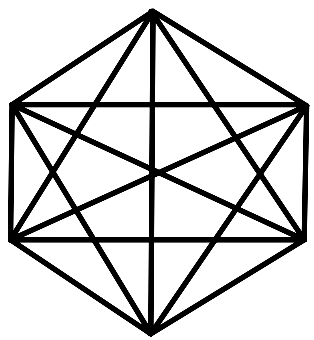
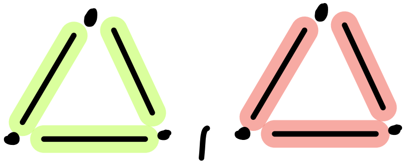
$G_1 :=$  $\longrightarrow$  YES

$K_6 =$  ?

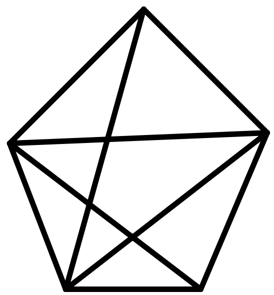
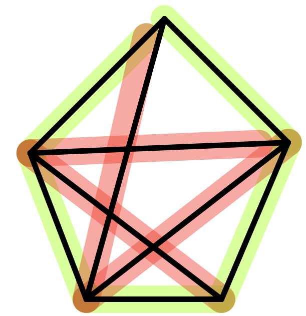
INTRODUCTION - EDGE COLORING PROBLEM

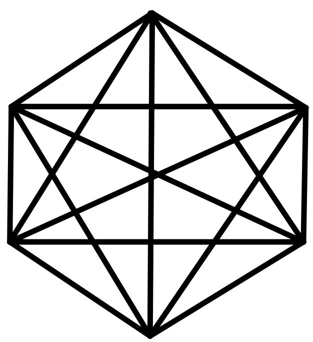
EXAMPLE

THIS PROBLEM IS NP-HARD

$F := \{$  $\}$

INPUT:

$G_1 :=$  →  YES

$K_6 =$  → NO: THE RAMSEY NUMBER OF (3,3) IS 6

OVERVIEW

BIG GOAL:

UNDERSTAND THE COMPLEXITY OF $\mathrm{Col}(\mathcal{F})$ FOR ALL $\mathcal{F}$

MAIN QUEST:

- PRESENT A STRATEGY INSPIRED BY THE ONE THAT CHARACTERIZED VERTEX COLORING PROBLEMS
- UNDERSTAND ON WHAT IT NEEDS TO WORK

SIDE QUEST:

- CAN IT SOLVE

$\mathcal{F} = \{$  $\}$?

OVERVIEW

PART I : THE STRATEGY

* PRESENT THE STRATEGY

* WHAT DOES IT RELY ON?

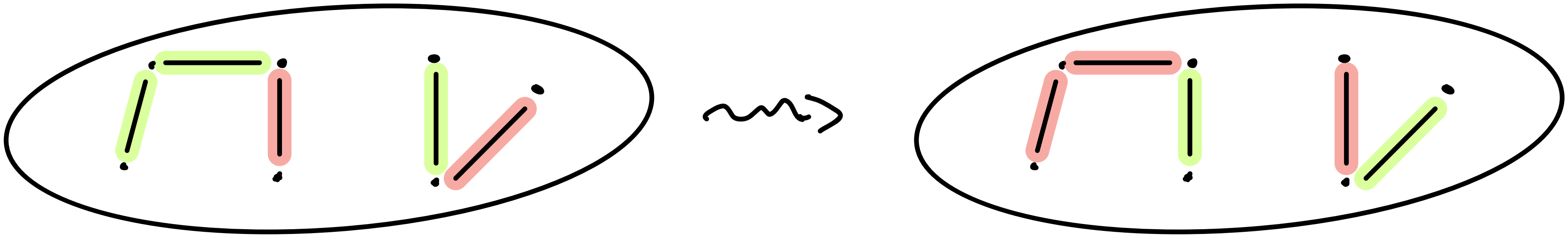
PART II: WHEN DOES THE STRATEGY WORK?

* PRESENT A CLASS OF PROBLEM WHERE IT DOES NOT WORK
* PRESENT A CLASS OF PROBLEM WHERE IT DOES WORK

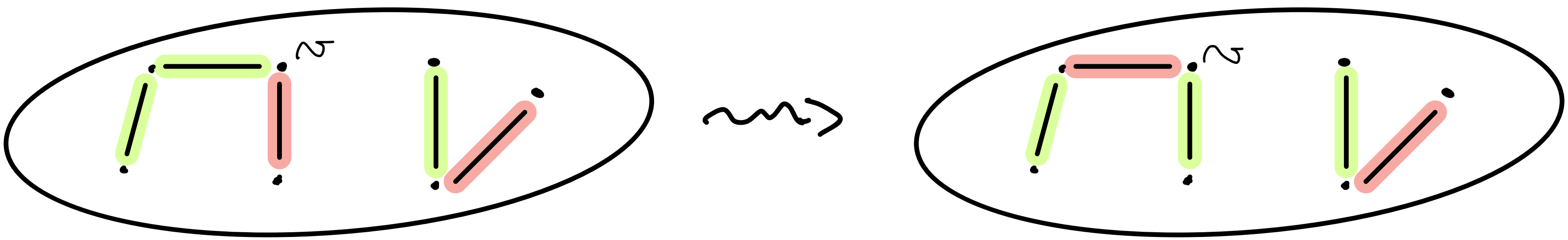
CONVENTIONS

1 USUALLY TWO COLORS $\{\bullet, \bullet\}$

2. FLIPPING

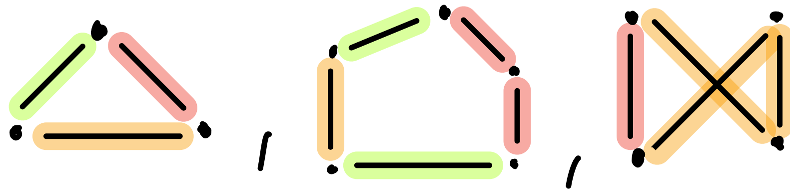


3. LOCAL FLIPPING ON A VERTEX v



INTRODUCTION - EXTENSION PROBLEM

FIX A SET OF COLORS = $\{$  $\}$

FIX $\mathcal{F} = \{$  $, \dots \}$

INPUT: A PARTIALLY COLORED GRAPH (G, α)

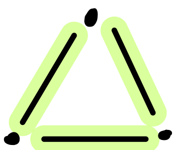
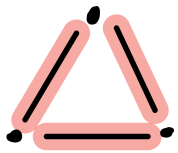
TASK: $\exists \xi : E(G) \longrightarrow$ COLORS SUCH THAT

* ξ EXTENDS α AND
* $\forall (F, \chi) \in \mathcal{F}$ $\quad (F, \chi) \underset{\text{HOMOMORPHISM}}{\not\longrightarrow} (G, \xi)$?

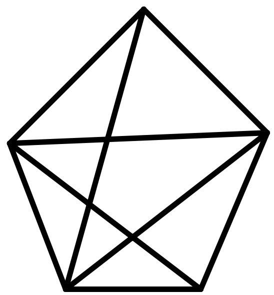
WE INDICATE SUCH PROBLEM $\mathrm{Ext}(\mathcal{F})$

INTRODUCTION - EXTENSION PROBLEM

EXAMPLE

$F := \{$  ,  $\}$

INPUT:

$(G, \alpha = \emptyset) :=$  ?

INTRODUCTION - EXTENSION PROBLEM

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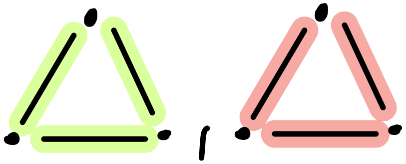
$F = \{$  $\}$

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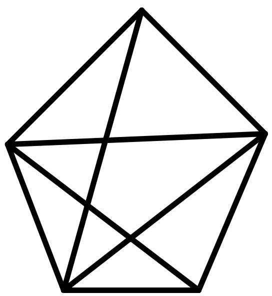
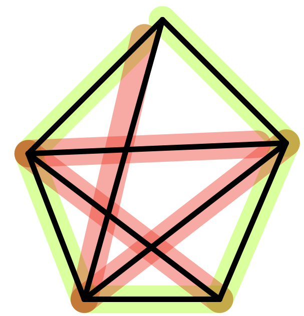
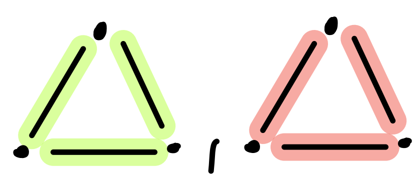
$(G, \alpha = \emptyset) :=$ 



YES



INTRODUCTION - EXTENSION PROBLEM

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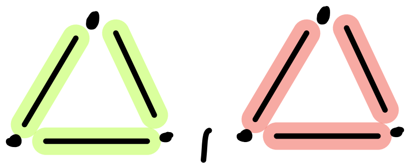
$F = \{$  $\}$

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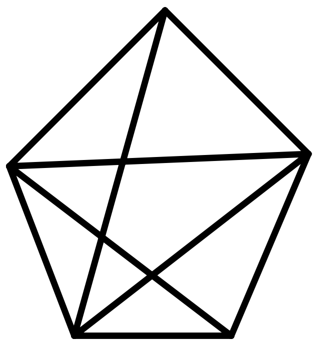
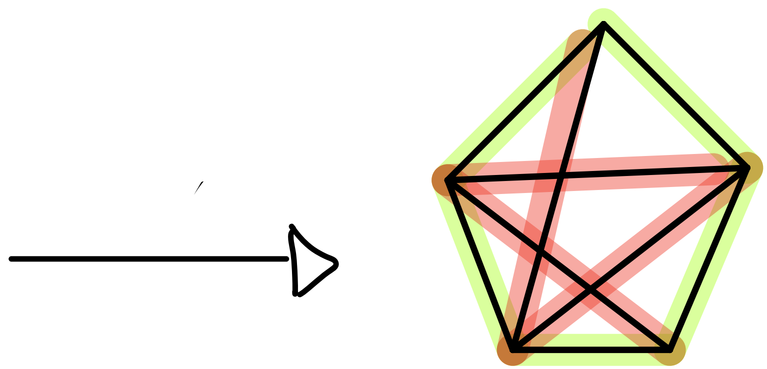
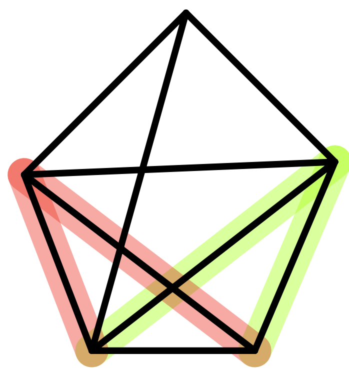
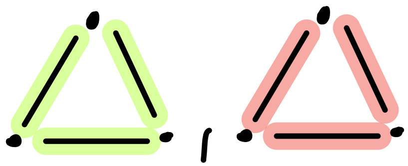
$(G, \alpha = \emptyset) :=$ 

 YES

$(G, \beta) :=$  ?

INTRODUCTION - EXTENSION PROBLEM

EXAMPLE

$F = \{$  $\}$

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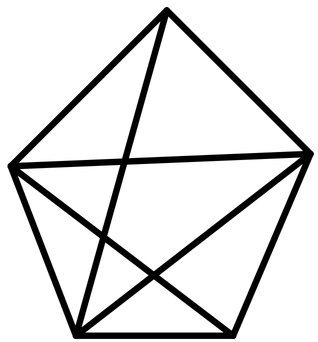
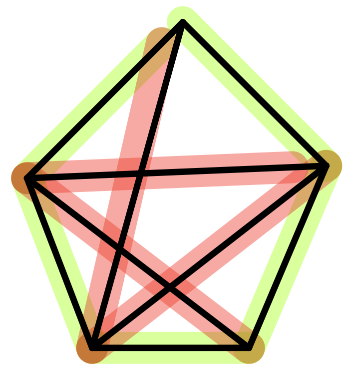
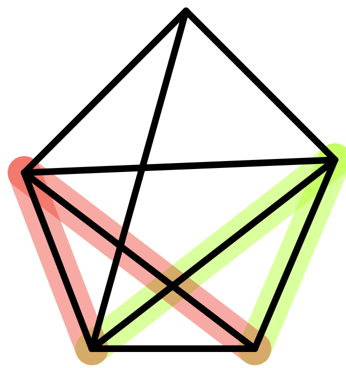
$(G, \alpha = \emptyset) :=$  $\longrightarrow$  YES

$(G, \beta) :=$  $\longrightarrow$ NO: THE BOTTOM EDGE

INTRODUCTION - CSP

FIX A SIGNATURE σ

FIX A RELATIONAL STRUCTURE F

INPUT: A STRUCTURE G IN THE SAME SIGNATURE

TASK: $\exists\, h : G \longrightarrow F$? WE INDICATE IT $CSP(F)$

HOMOMORPHISM

INTRODUCTION - CSP

FIX A RELATIONAL STRUCTURE F

INPUT: A STRUCTURE G IN THE SAME SIGNATURE

TASK: $\exists h : G \xrightarrow[\text{HOMOMORPHISM}]{} F$? WE INDICATE IT $CSP(F)$

GIVEN $\mathcal{F}$ WE DEFINE

- $G_{\mathcal{F}} = \left(\text{COLORS}, (R_F)_{(F,x) \in \mathcal{F}}, (C_i)_{i \in \text{COLORS}} \right)$
- R_F IS $|E(F)|$-ARY AND CONTAINS THE VALID COLORING OF F,
- C_i IS UNARY AND $C_i = \{i\}$

INTRODUCTION - FINITE CSP

GIVEN $\mathcal{F}$ WE DEFINE

- $G_{\mathcal{F}} = \left(\text{COLORS}, (R_F)_{(F,\chi)\in\mathcal{F}}, (C_i)_{i\in\text{COLORS}}\right)$
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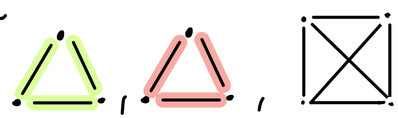
EXAMPLE

$\mathcal{F} = \{$  $\}$

$G_{\mathcal{F}} = \left(\{\bullet,\bullet\}, R_{K_3}, R_{K_4}, C_{\bullet}, C_{\bullet}\right)$

$R_{K_3} = \{\bullet,\bullet\}^3 - \{(\bullet,\bullet,\bullet);(\bullet,\bullet,\bullet)\}$

$R_{K_4} = \emptyset$

$C_{\bullet} = \{\bullet\}$

$C_{\bullet} = \{\bullet\}$

INTRODUCTION - FINITE CSP

GIVEN $\mathcal{F}$ WE DEFINE

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PROPOSITION

$$\mathrm{Ext}(\mathcal{F}) \leq_p \mathrm{CSP}(G_{\mathcal{F}})$$

PART I

THE STRATEGY

WHAT'S THE STRATEGY?

GROUND IDEA: LIFT THE CHARACTERIZATION FOR FINITE DOMAIN CSP.

GOAL: $\mathrm{Col}(\mathcal{F}) \approx_P \mathrm{CSP}(G_{\mathcal{F}})$

WHAT'S THE STRATEGY?

STEP 0 $\mathrm{Col}(\mathcal{F}) \leq_p \mathrm{Ext}(\mathcal{F})$ AND $\mathrm{Ext}(\mathcal{F}) \leq_p \mathrm{CSP}(G_{\mathcal{F}})$

GOAL: $\mathrm{Col}(\mathcal{F}) \approx_p \mathrm{CSP}(G_{\mathcal{F}})$

WHAT'S THE STRATEGY?

STEP 0 $Col(\mathcal{F}) \leq_p Ext(\mathcal{F})$ AND $Ext(\mathcal{F}) \leq_p CSP(G_{\mathcal{F}})$

STEP 1 FIND A NICE STRUCTURE $H_{\mathcal{F}}$: $CSP(H_{\mathcal{F}}) \approx_p Col(\mathcal{F})$

GOAL: $Col(\mathcal{F}) \approx_p CSP(G_{\mathcal{F}})$

WHAT'S THE STRATEGY?

STEP 0 $\mathrm{Col}(\mathcal{F}) \leq_p \mathrm{Ext}(\mathcal{F})$ AND $\mathrm{Ext}(\mathcal{F}) \leq_p \mathrm{CSP}(G_{\mathcal{F}})$

STEP 1 FIND A NICE STRUCTURES $H_{\mathcal{F}}$: $\mathrm{CSP}(H_{\mathcal{F}}) \approx_p \mathrm{Col}(\mathcal{F})$

STEP 2 FIND A GADGET REDUCTION:

$$\mathrm{CSP}(H_{\mathcal{F}}) \geq_p \mathrm{Ext}(\mathcal{F})$$

GOAL: $\mathrm{Col}(\mathcal{F}) \approx_p \mathrm{CSP}(G_{\mathcal{F}})$

WHAT'S THE STRATEGY?

STEP 0 $\mathrm{Col}(\mathcal{F}) \leq_p \mathrm{Ext}(\mathcal{F})$ AND $\mathrm{Ext}(\mathcal{F}) \leq_p \mathrm{CSP}(G_{\mathcal{F}})$

STEP 1 FIND A NICE STRUCTURES $\mathcal{H}_{\mathcal{F}}$: $\mathrm{CSP}(\mathcal{H}_{\mathcal{F}}) \approx_p \mathrm{Col}(\mathcal{F})$

STEP 2 FIND A GADGET REDUCTION:

$$\mathrm{CSP}(\mathcal{H}_{\mathcal{F}}) \geq_p \mathrm{Ext}(\mathcal{F})$$

STEP 3 FIND A GADGET REDUCTION:

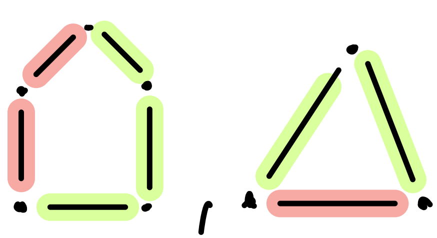
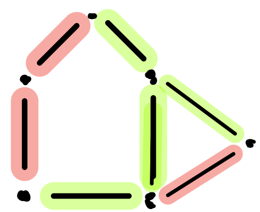
$$\mathrm{Ext}(\mathcal{F}) \geq_p \mathrm{CSP}(G_{\mathcal{F}})$$

GOAL: $\mathrm{Col}(\mathcal{F}) \approx_p \mathrm{CSP}(G_{\mathcal{F}})$

ASSUMPTION

WE CONSIDER TO WORK WITH A SET $\mathcal{F}$ SUCH THAT THE SET OF COLORED GRAPHS WHICH ARE VALID FOR $\mathcal{F}$ IS CLOSED UNDER EDGE-AMALGAMATION

EXAMPLE

 VALID FOR $\mathcal{F}$ $\Longrightarrow$  VALID FOR $\mathcal{F}$

LEMMA

FOR EVERY FINITE SET OF COLORED GRAPHS $\mathcal{F}$ THERE EXISTS $\mathcal{F}'$ SUCH THAT

1. $\mathrm{Col}(\mathcal{F})$ IS EQUIVALENT TO $\mathrm{Col}(\mathcal{F}')$
2. $\mathcal{F}'$ HAS EDGE-AMALGAMATION

STEP 1

INFINITE-DOMAIN CSP

STEP 1

THEOREM (BODIRSKY, KNÄUER, STARKE)

GIVEN A FINITE SET OF COLORED GRAPHS $\mathcal{F}$ THERE EXISTS A "NICE" STRUCTURE $H_{\mathcal{F}}$ SUCH THAT

$$\mathrm{Col}(\mathcal{F}) \text{ IS } \underline{\text{EQUIVALENT}} \text{ TO } \mathrm{CSP}(H_{\mathcal{F}})$$

STEP 1

THEOREM (BODIRSKY, KNÄUER, STARKE)

GIVEN A FINITE SET OF COLORED GRAPHS $\mathcal{F}$ THERE EXISTS A "NICE" STRUCTURE $H_{\mathcal{F}}$ SUCH THAT

$$\mathrm{Col}(\mathcal{F}) \text{ IS } \underline{\text{EQUIVALENT}} \text{ TO } \mathrm{CSP}(H_{\mathcal{F}})$$

WHAT DOES "NICE" MEAN?

(* IT BELONGS TO THE SCOPE OF THE BODIRSKY-PINSKER CONJECTURE FOR INFINITE-DOMAIN CSP)

* MORE IMPORTANTLY: THIS STRUCTURE CAN BE EXPANDED WITH CONSTANTS WITHOUT INCREASING THE COMPLEXITY

STEP 2

COLOR DETERMINERS

STEP 2

GOAL : $CSP(H_F) \geq_p Ext(F)$

OBSERVATION

$$Col(F) \neq Ext(F)$$

INPUT

STEP 2

GOAL: $CSP(H_F) \geq_p Ext(F)$

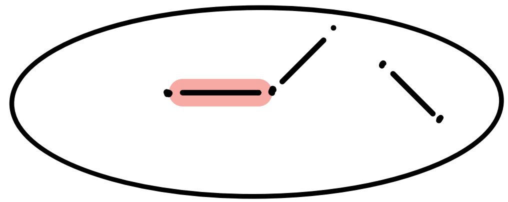
OBSERVATION

$$Col(F) \neq Ext(F)$$

INPUT

IDEA

LET'S TAKE AN INSTANCE FOR $Ext(F)$ (H, α) ::



STEP 2

GOAL: $CSP(H_F) \geq_p Ext(F)$

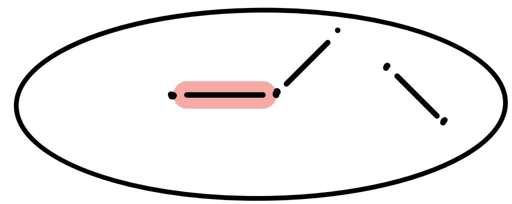
OBSERVATION

$$Col(F) \neq Ext(F)$$

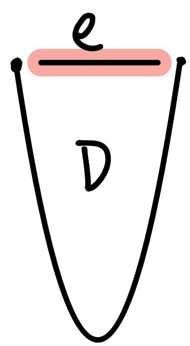
INPUT

IDEA

LET'S TAKE AN INSTANCE FOR $Ext(F)$ (H, α) ::



YES INSTANCE FOR $Col(F)$



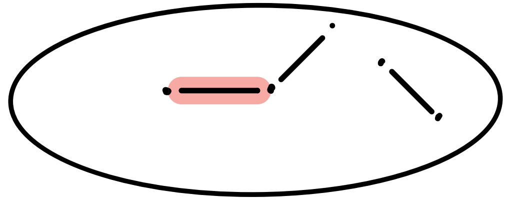
WITH $e \in E(D)$ SUCH THAT

$\forall \chi : E(D) \to$ COLORS

χ IS VALID $\Rightarrow$ $\chi(e) =$

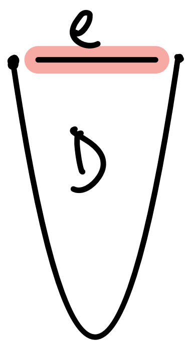
STEP 2 GOAL: $CSP(H_F) \geq_p Ext(F)$

IDEA

LET'S TAKE AN INSTANCE FOR $Ext(F)$ $(H, \alpha) ::$ 

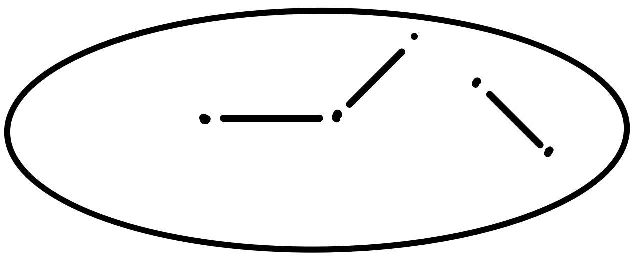
YES INSTANCE FOR $Col(F)$

WITH $e \in E(D)$ SUCH THAT



$\forall \chi : E(D) \longrightarrow$ COLORS

χ IS VALID $\Rightarrow$ $\chi(e) =$

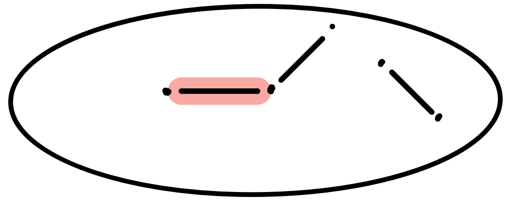
$H =$ 

STEP 2

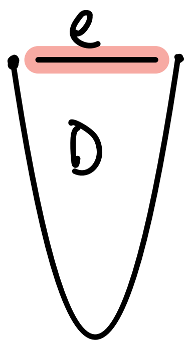
GOAL: $CSP(H_F) \geq_p Ext(F)$

IDEA

LET'S TAKE AN INSTANCE FOR $Ext(F)$ $(H, \alpha) :=$



YES INSTANCE FOR $Col(F)$

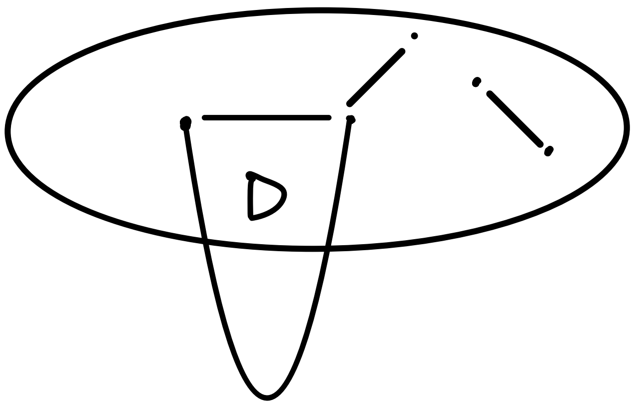


WITH $e \in E(D)$ SUCH THAT

$\forall \chi : E(D) \longrightarrow$ COLORS

χ IS VALID $\Rightarrow \chi(e) =$

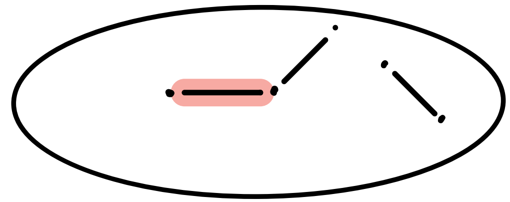
$H' :=$



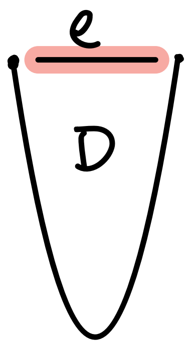
STEP 2 GOAL: $CSP(H_F) \geq_p Ext(F)$

IDEA

LET'S TAKE AN INSTANCE FOR $Ext(F)$ $(H, \alpha) :=$



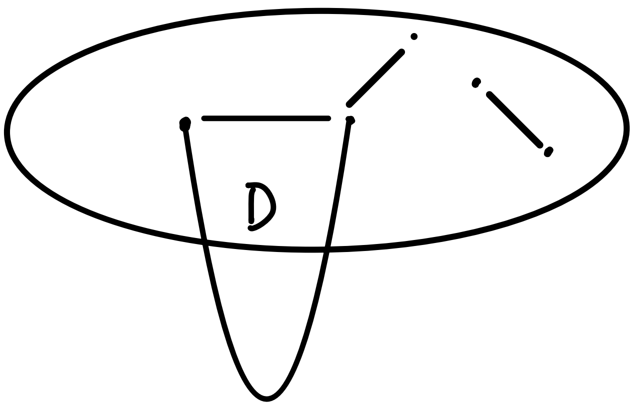
YES INSTANCE FOR $Col(F)$ WITH $e \in E(D)$ SUCH THAT



$\forall \chi : E(D) \longrightarrow$ COLORS

χ IS VALID $\Rightarrow$ $\chi(e) =$

$H' :=$



PROPOSITION

(H, α) IS A YES INSTANCE FOR $Ext(F)$

IFF

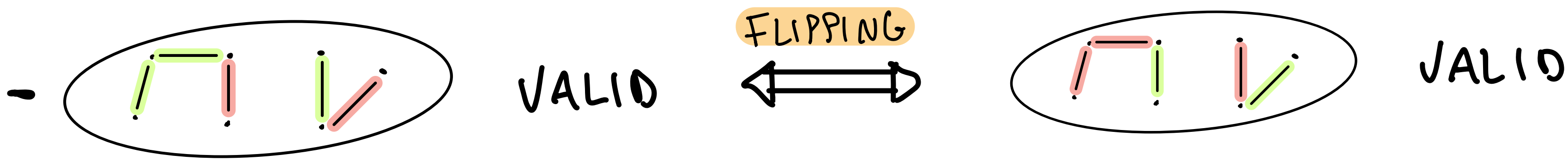
H' IS A YES INSTANCE FOR $Col(F)$

STEP 2 GOAL: $CSP(H_F) \geq_p Ext(F)$

BAD NEWS THESE GADGETS DO NOT EXIST OFTEN, NOT EVEN FOR OUR SIDE QUEST

PROPOSITION

IF F IS CLOSED UNDER FLIPPING THEN

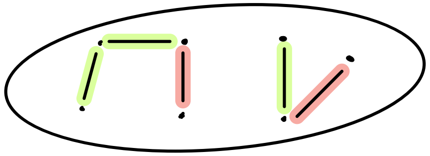
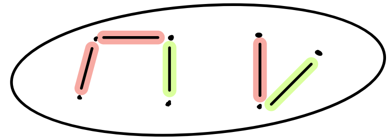


- WE DO NOT HAVE THESE GADGETS

STEP 2   GOAL: $CSP(H_F) \geq_p Ext(F)$

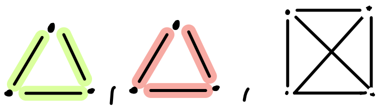
PROPOSITION

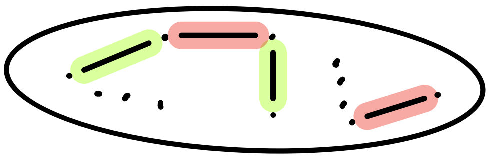
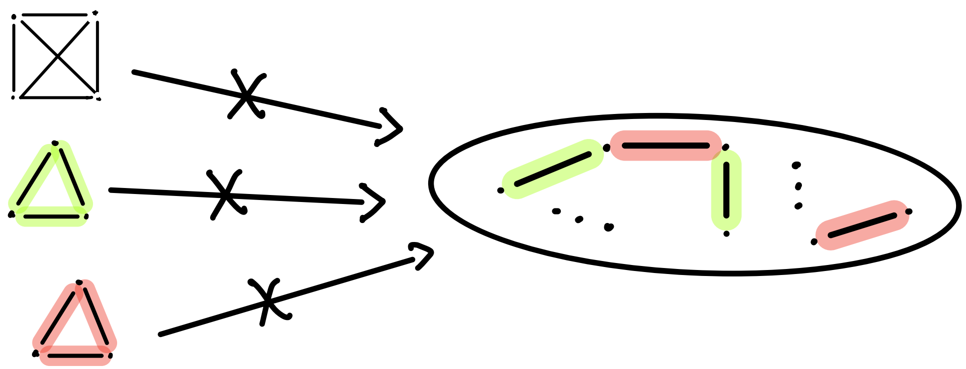
IF F IS CLOSED UNDER FLIPPING THEN

-  VALID $\xLeftrightarrow{\text{FLIPPING}}$  VALID

- WE DO NOT HAVE THESE GADGETS

EXAMPLE

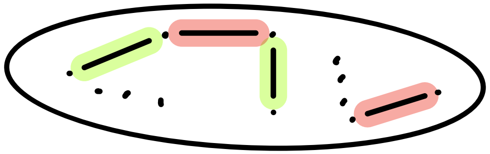
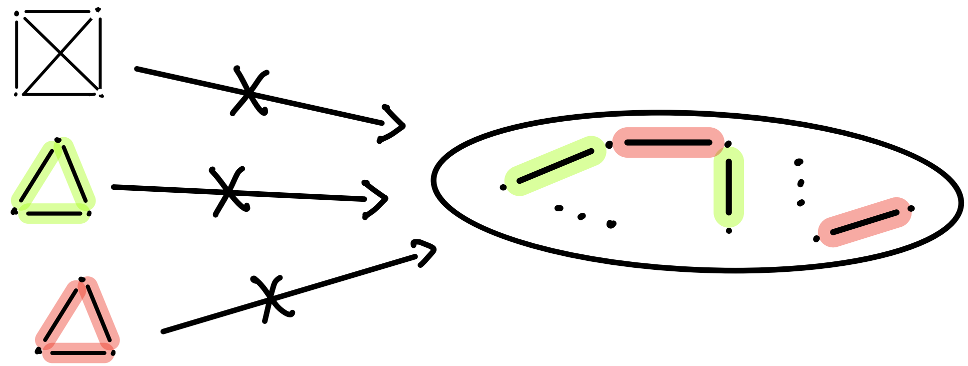
$F = \{$  $\}$

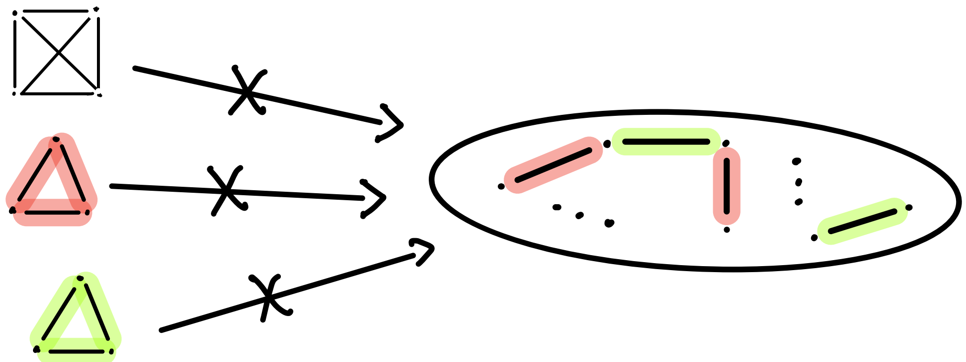
(G, x)  IS VALID $\Leftrightarrow$ 

STEP 2   GOAL: $CSP(H_F) \geq_p Ext(F)$

EXAMPLE

$F = $ 

(G, x)  IS VALID $\iff$ 

$\iff$ FLIPPING 

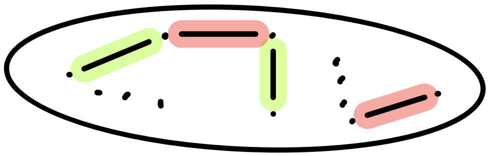
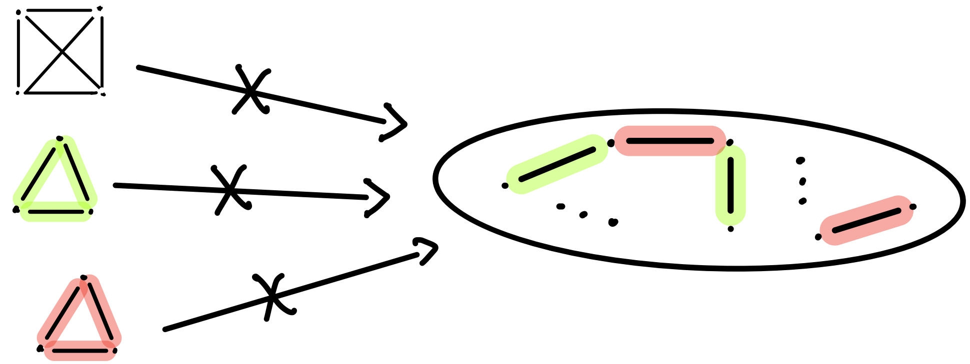
OBSERVATION

FLIPPING THE COLORS CANNOT GENERATE A MONOCHROMATIC K_3 OR A K_4

STEP 2   GOAL: $CSP(H_F) \geq_p Ext(F)$

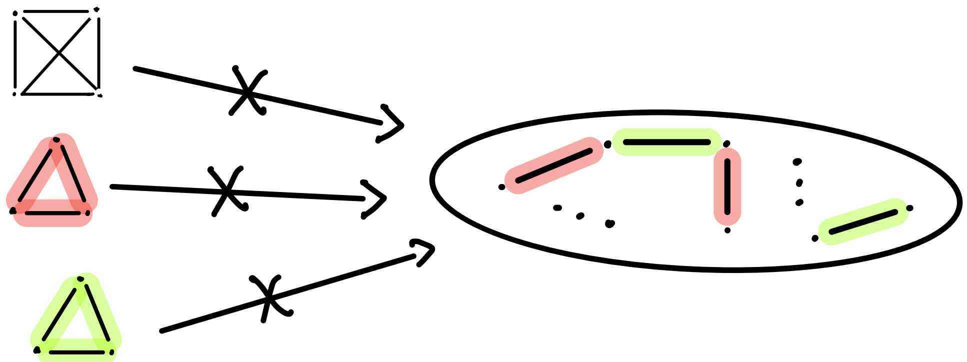
EXAMPLE

$F = \{$  $\}$

(G, x)  IS VALID $\Longleftrightarrow$ 

$\Longleftrightarrow$

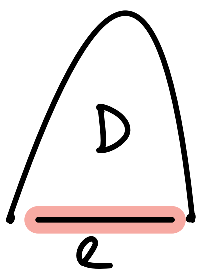
FLIPPING

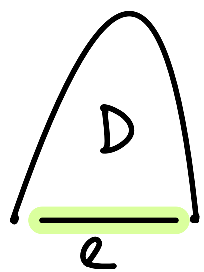


OBSERVATION

FLIPPING THE COLORS CANNOT GENERATE A MONOCHROMATIC K_3 OR A K_4

IN PARTICULAR

$(D, \xi) :=$ 

VALID $\Longleftrightarrow$ FLIPPING 

VALID

HENCE THE DESIRED GADGET DOES NOT EXIST.

STEP 2 GOAL: $CSP(H_F) \geq_p Ext(F)$

BUT WE SAW BEFORE THAT THE H_F ALLOWS US TO USE A BOUNDED AMOUNT OF CONSTANTS.

STEP 2 GOAL: $CSP(H_F) \geq_p Ext(F)$

BUT WE SAW BEFORE THAT THE INFINITE DOMAIN CSP ALLOWS US TO USE A BOUNDED AMOUNT OF CONSTANTS.

IDEA 2.0

CONSTANTS $\xleftrightarrow{\text{ACTING AS}}$ COLORED EDGES

WE COULD USE BOUNDED AMOUNT OF COLORED EDGES

STEP 2   GOAL: $CSP(H_F) \geq_p Ext(F)$

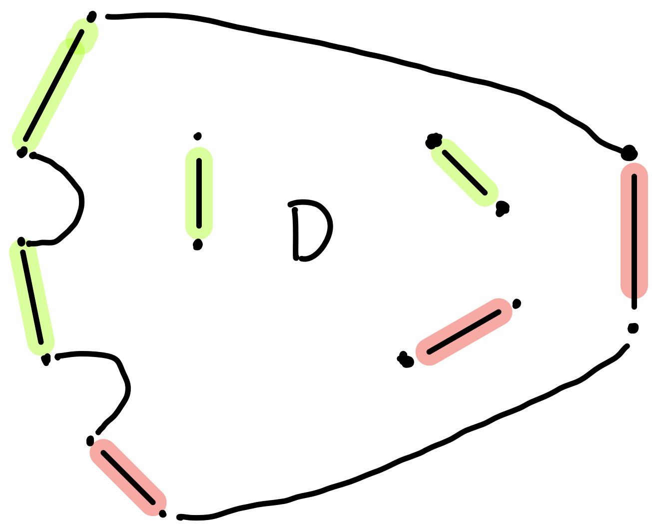
IDEA 2.0

CONSTANTS $\xleftrightarrow{\text{ACTING AS}}$ COLORED EDGES

WE COULD USE BOUNDED AMOUNT OF COLORED EDGES

SO WE CAN LOOK FOR

- YES INSTANCES
- WITH α VALID COLORING



STEP 2 GOAL: $CSP(H_F) \geq_p Ext(F)$

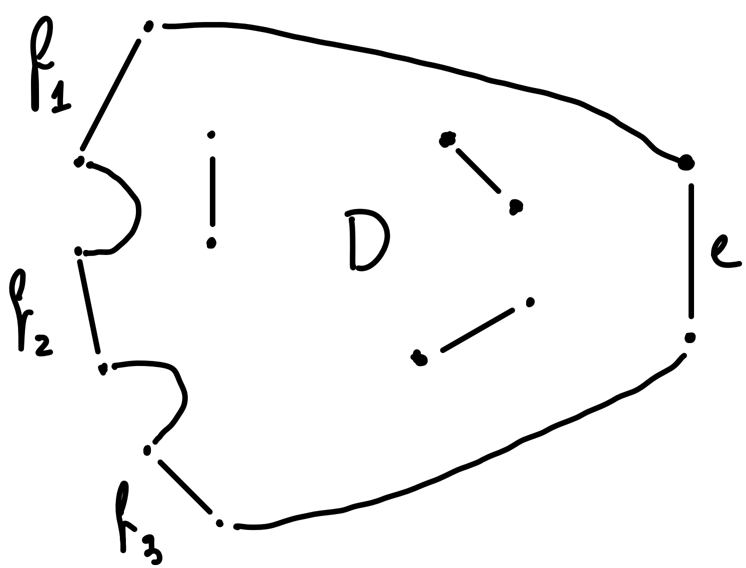
IDEA 2.0

$$\text{CONSTANTS} \xleftrightarrow{\text{ACTING AS}} \text{COLORED EDGES}$$

WE COULD USE BOUNDED AMOUNT OF COLORED EDGES

SO WE CAN LOOK FOR

- YES INSTANCES $\propto$ VALID COLORING
- $f_1, f_2, f_3, e \in E(D)$



STEP 2

GOAL: $CSP(H_F) \geq_p Ext(F)$

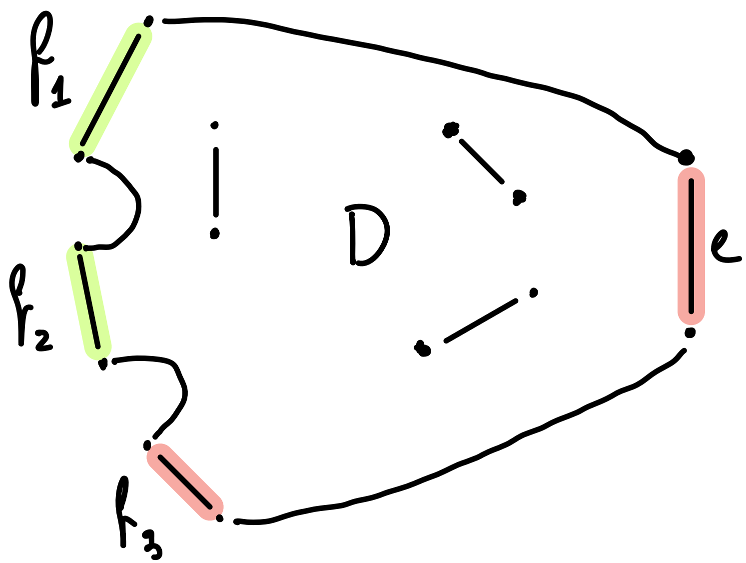
IDEA 2.0

CONSTANTS $\xleftrightarrow{\text{ACTING AS}}$ COLORED EDGES

WE COULD USE BOUNDED AMOUNT OF COLORED EDGES

SO WE CAN LOOK FOR

- YES INSTANCES
  α VALID COLORING
- $f_1, f_2, f_3, e \in E(D)$
- $\forall \chi : E(D) \to$ COLORS
  IF χ IS VALID AND
  $\chi_{|f_1, f_2, f_3} = \alpha$
  THEN $\chi(e) =$



STEP 2 GOAL: $CSP(H_F) \geq_p Ext(F)$

IDEA 2.0

$$\text{CONSTANTS} \xleftrightarrow{\text{ACTING AS}} \text{COLORED EDGES}$$

WE COULD USE BOUNDED AMOUNT OF COLORED EDGES

SO WE CAN LOOK FOR

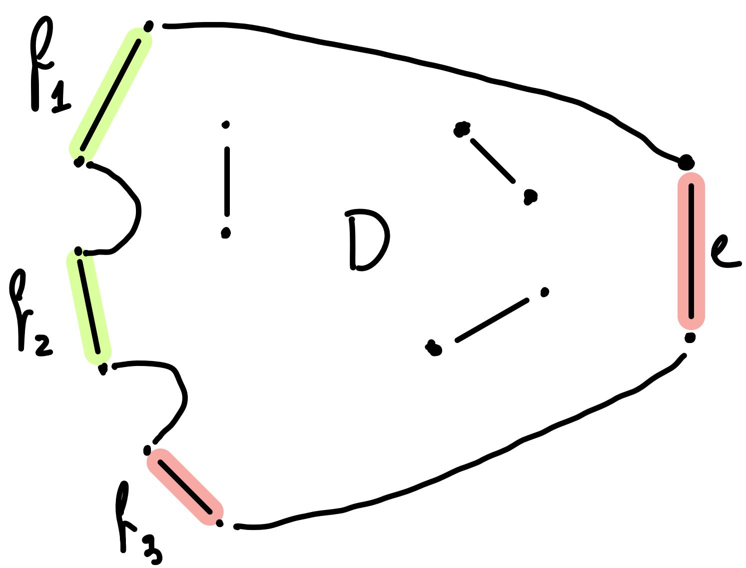
- YES INSTANCES
  ∝ VALID COLORING
- $f_1, f_2, f_3, e \in E(D)$
- $\forall \chi : E(D) \to \text{COLORS}$

  IF χ IS VALID AND

  $\chi(f_1) =$ ● $\chi(f_2) =$ ● $\chi(f_3) =$ ●

  $\Downarrow$

  $\chi(e) =$ ●



REMARK

* THIS ALLOWS e TO BE ALSO COLORED IN ●
* TO BE USEFUL IN PRACTICE $dist(e, f_i) \gg 0$

WE CALL THEM COLOR DETERMINERS

STEP 3

COLOR EQUALITY GADGET

STEP 3

GOAL: $CSP(G_F) \leq_p Ext(F)$

OBSERVATION

$CSP(G_F)$	RELATIONS	COORDINATES OF R_F
$Ext(F)$	UNCOLORED OBSTRUCTIONS	EDGE OF F

STEP 3

GOAL: $CSP(G_F) \leq_p Ext(F)$

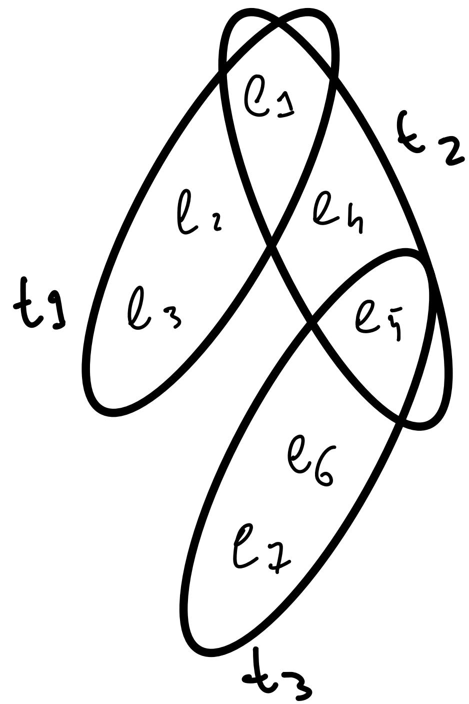
OBSERVATION

$CSP(G_F)$	RELATIONS	COORDINATO OF R_F
$Ext(F)$	UNCOLORED OBSTRUCTIONS	AN EDGE OF F

IDEA

$F = \{$  $\}$

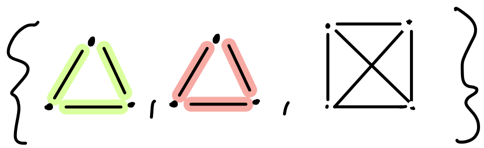
GIVEN AN INSTANCE FOR $CSP(G_F)$



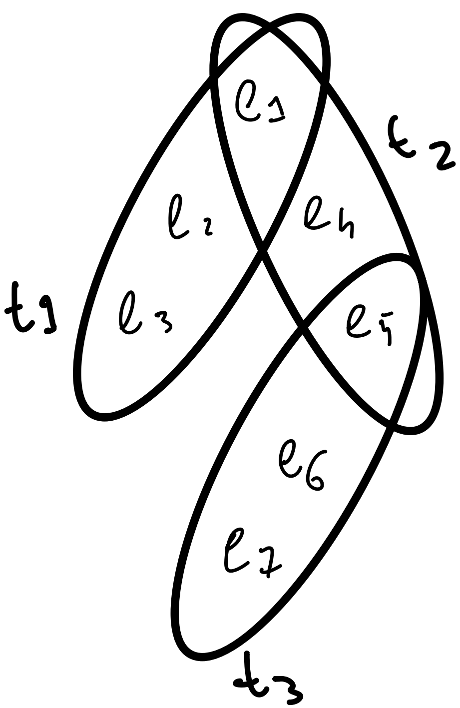
STEP 3 GOAL: $CSP(G_F) \leq_p Ext(F)$

OBSERVATION

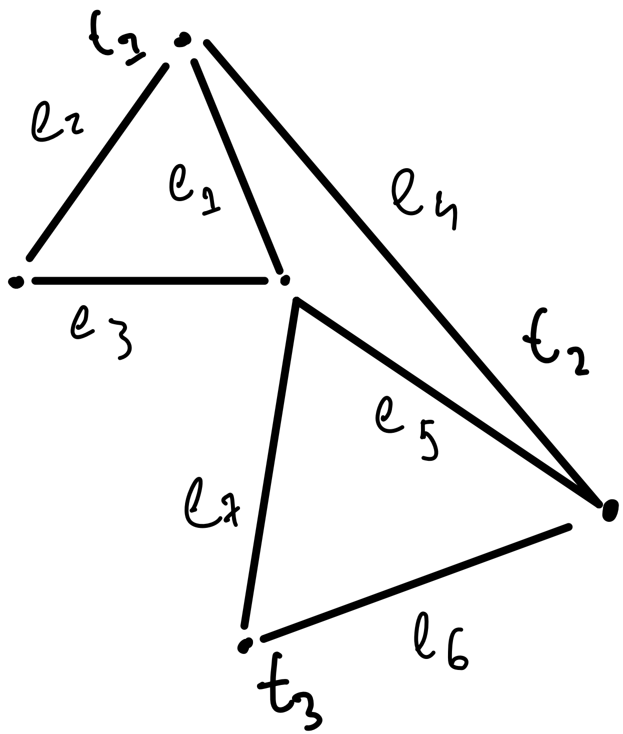
$CSP(G_F)$	RELATIONS	COORDINATE OF R_F
$Ext(F)$	UNCOLORED OBSTRUCTIONS	AN EDGE OF F

IDEA $F = $ 

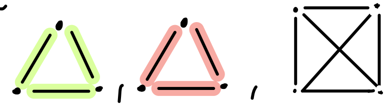
GIVEN AN INSTANCE FOR $CSP(G_F)$



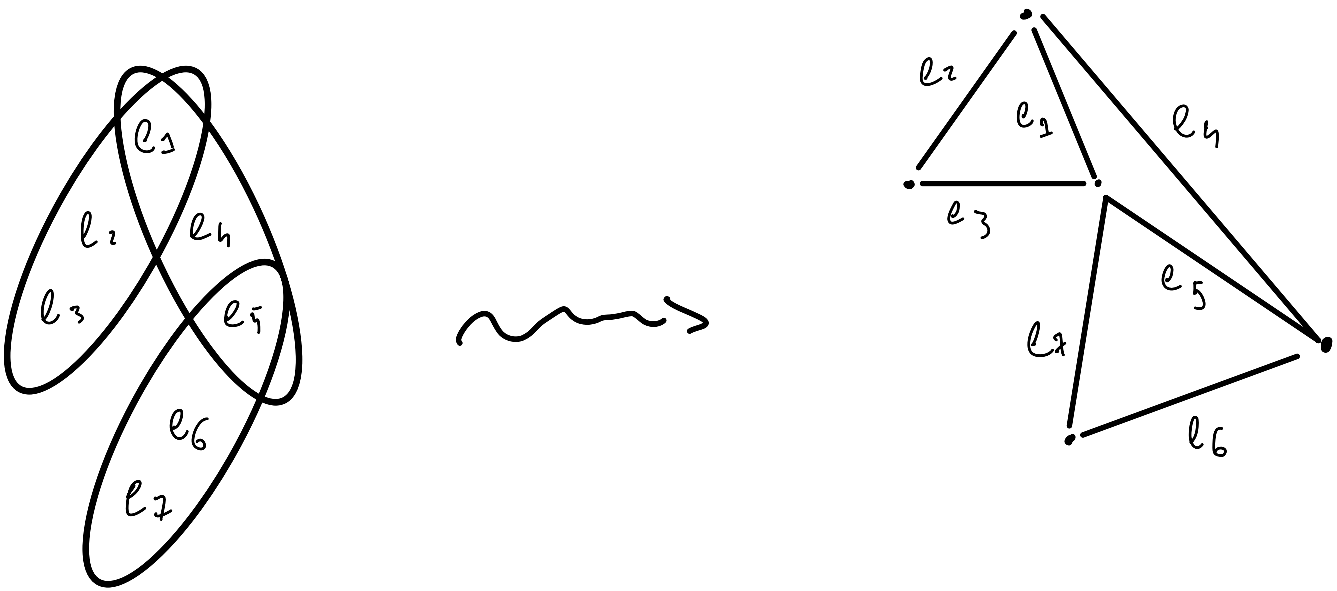
OBSERVATION
$\rightsquigarrow$



STEP 3 GOAL: $CSP(G_F) \leq_p Ext(F)$

IDEA $F = \{$  $\}$

GIVEN AN INSTANCE FOR $CSP(G_F)$



PROBLEM

THIS CAN PRODUCE NEW TRIANGLES!

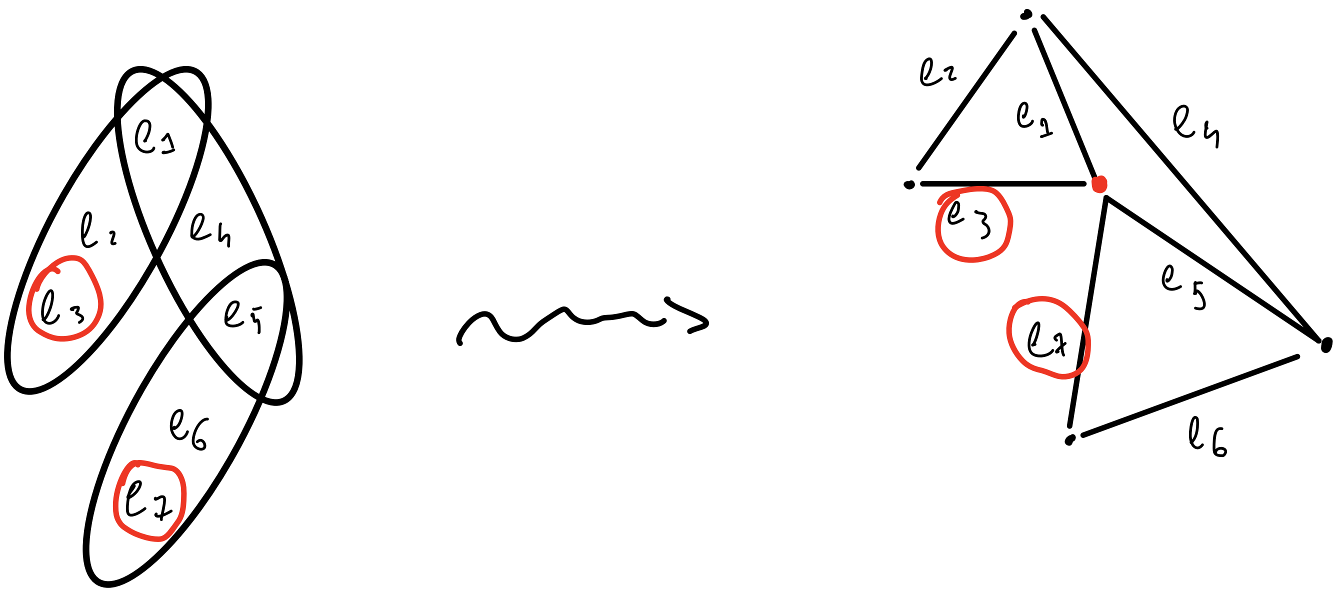
STEP 3

GOAL: $CSP(G_F) \leq_p Ext(F)$

IDEA

$F = \{$  $\}$

GIVEN AN INSTANCE FOR $CSP(G_F)$



PROBLEM

THIS CAN PRODUCE NEW TRIANGLES!

OBSERVATION

e_3, e_7 AFTER THE OPERATION SHARE A VERTEX

WE HAVE A PROBLEM OF DISTANCE

STEP 3

GOAL: $CSP(G_F) \leq_p Ext(F)$

BUT: IF WE COULD USE A YES INSTANCE EQ ::= e | EQ | f SUCH THAT $\forall \chi : E(EQ) \to$ COLORS

$$\chi \text{ VALID} \Rightarrow \chi(e) = \chi(f)$$

DEFINITION

WE CALL THE TRIPLE (EQ, e, f) A COLOR-EQUALITY GADGET.

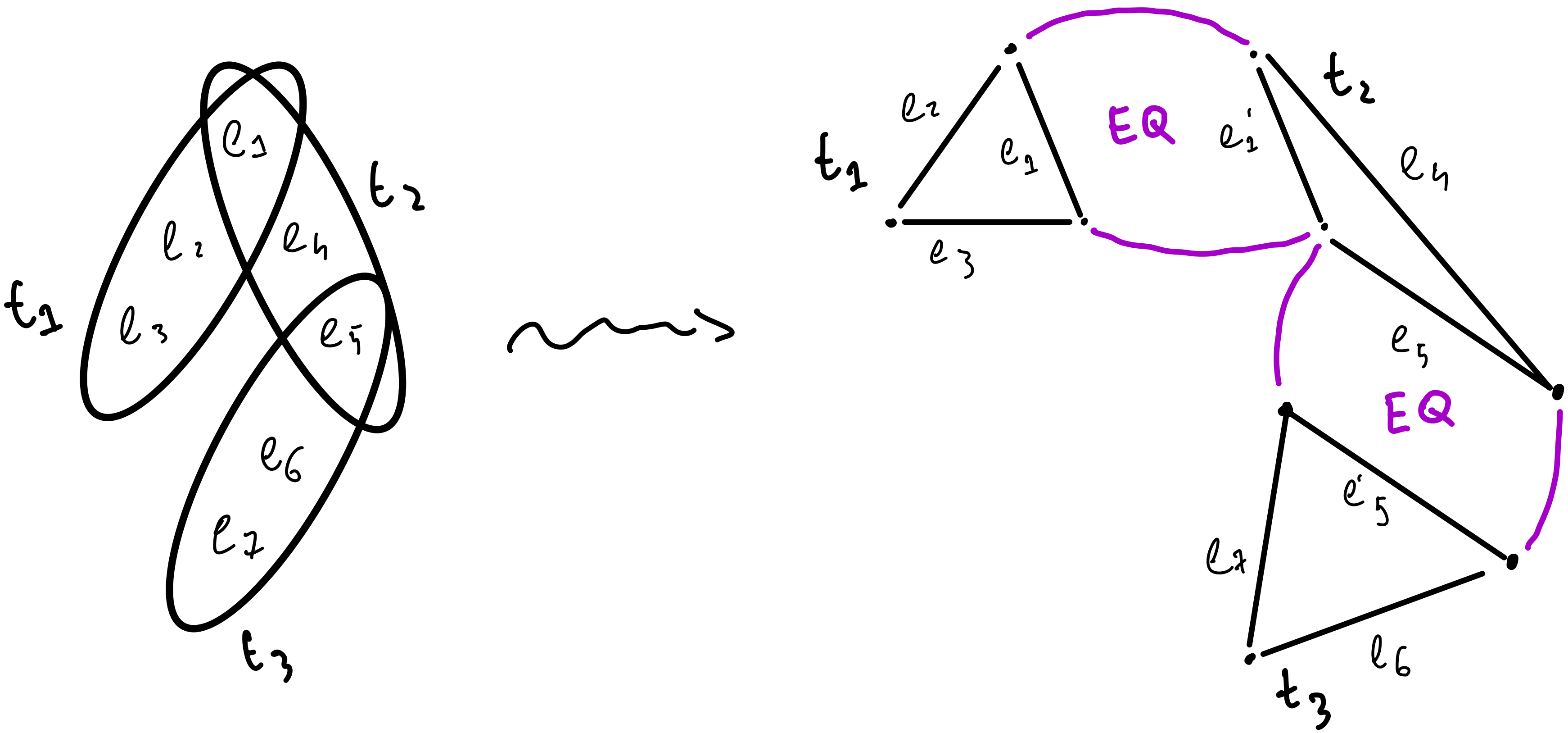
STEP 3

GOAL: $CSP(G_F) \leq_p Ext(F)$

BUT: IF WE COULD USE A YES INSTANCE EQ ::= e | EQ | f SUCH THAT $\forall \chi : E(EQ) \to$ COLORS

$$\chi \text{ VALID } \Rightarrow \chi(e) = \chi(f)$$

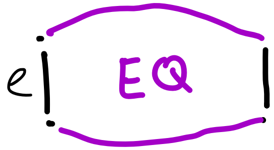
THEN WE WOULD FIX THIS PROBLEM OF DISTANCE



STEP 3

GOAL: $CSP(G_F) \le_p Ext(F)$

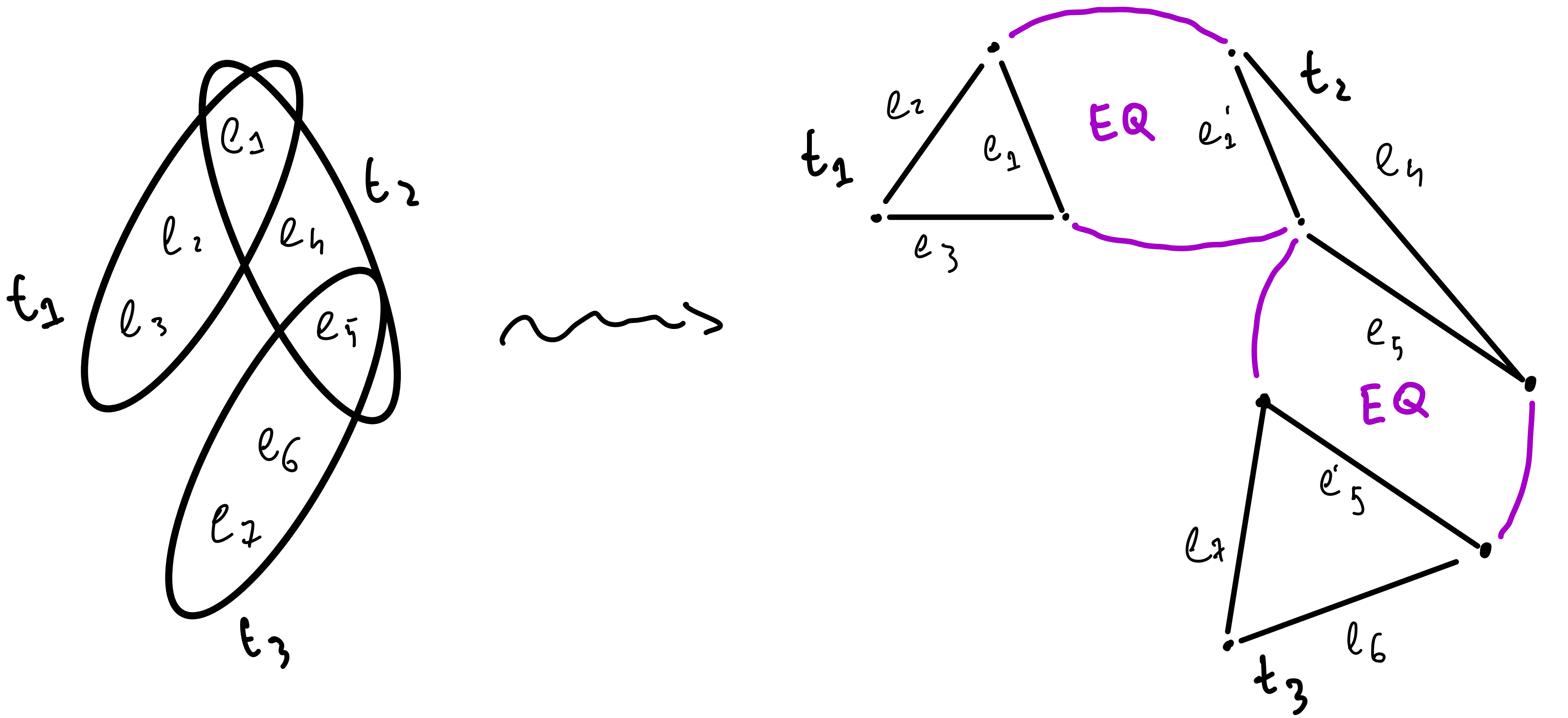
BUT: IF WE COULD USE A YES INSTANCE EQ ::=



SUCH THAT $\forall \chi : E(EQ) \to$ COLORS

$$\chi \text{ VALID} \implies \chi(e) = \chi(f)$$

THEN WE WOULD FIX THIS PROBLEM OF DISTANCE



THIS DOES NOT PRODUCE NEW TRIANGLES, INDEED THIS WORKS!!

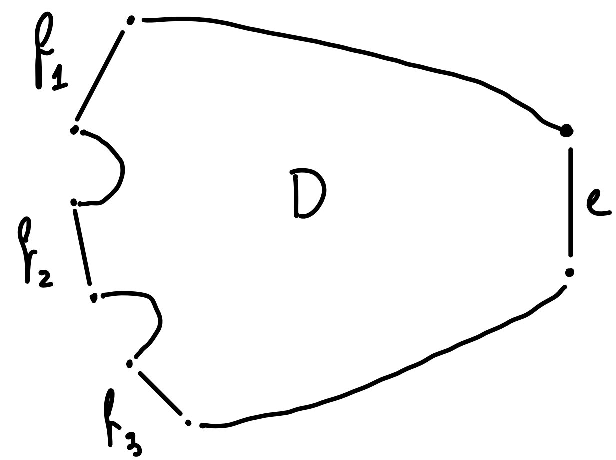
PART II

WHEN CAN WE FIND THESE GADGETS?

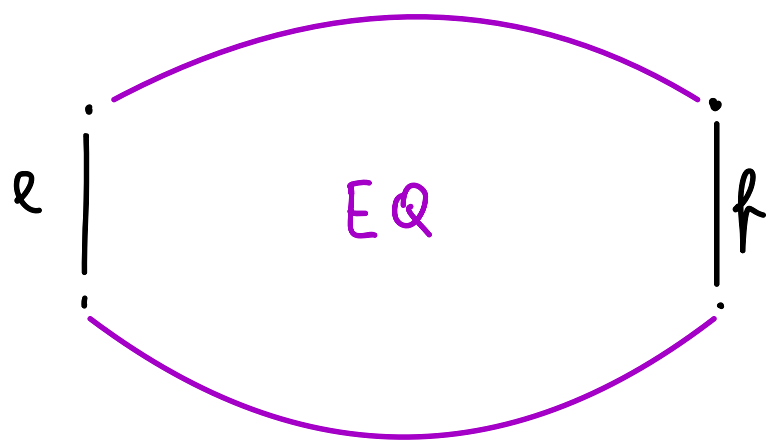
PART II

THIS STRATEGY USES 2 TYPES OF GADGETS

1. COLOR-DETERMINERS



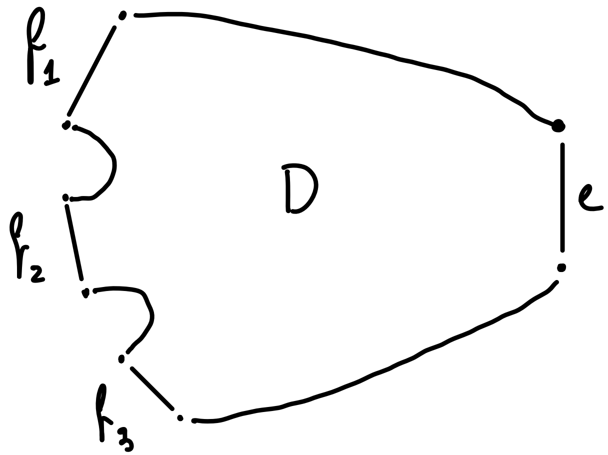
2. COLOR-EQUALITY



PART II

THIS STRATEGY USES 2 TYPES OF GADGETS

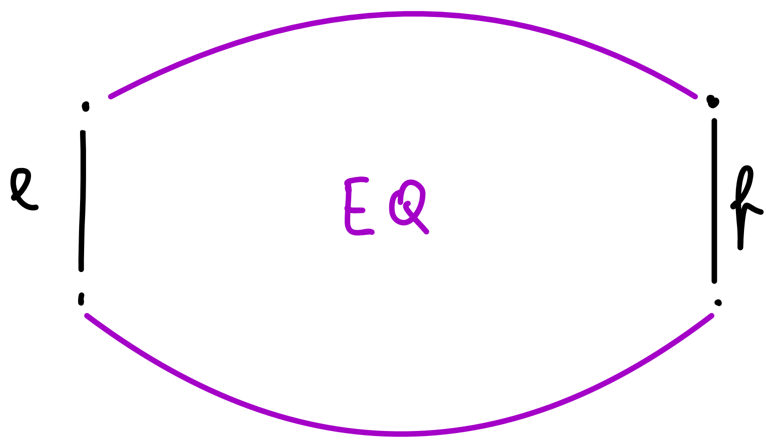
1. COLOR-DETERMINER



QUESTIONS

- DO THEY EXIST FOR ANY F?

2. COLOR-EQUALITY



DO THEY EXIST FOR ANY F?

SHORT ANSWER: NO

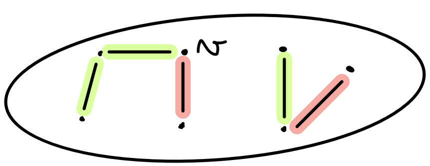
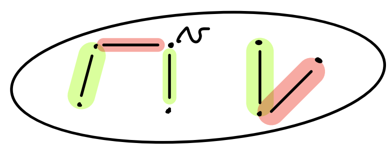
DO THEY EXIST FOR ANY F?

SHORT ANSWER: NO

LONGER ANSWER:

PROPOSITION

IF F IS CLOSED UNDER LOCAL FLIPPING THEN

 VALID $\rightsquigarrow$  VALID

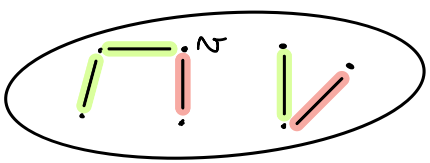
DO THEY EXIST FOR ANY F?

SHORT ANSWER: NO

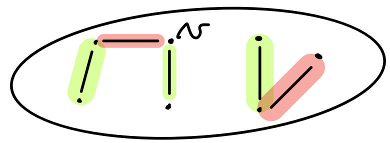
LONGER ANSWER:

PROPOSITION

IF F IS CLOSED UNDER LOCAL FLIPPING THEN

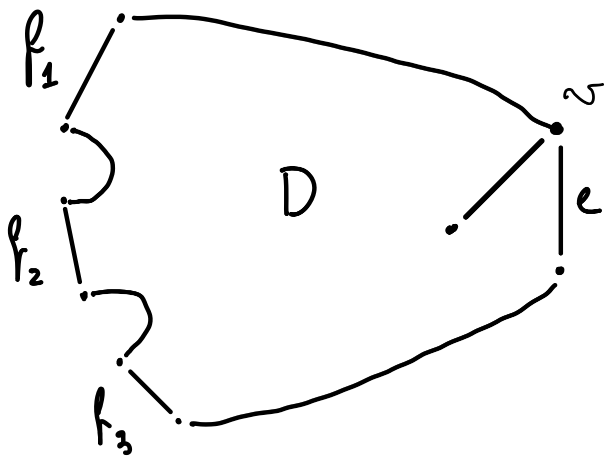


VALID $\rightsquigarrow$



VALID

TAKE A COLOR DETERMINER (D, f_i, e)

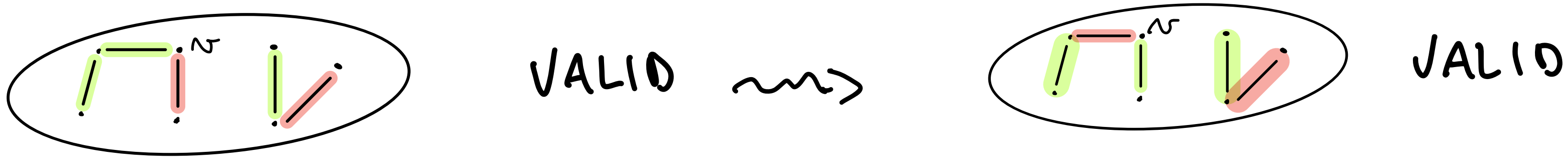


$\text{dist}(f_i, e) \gg 0$

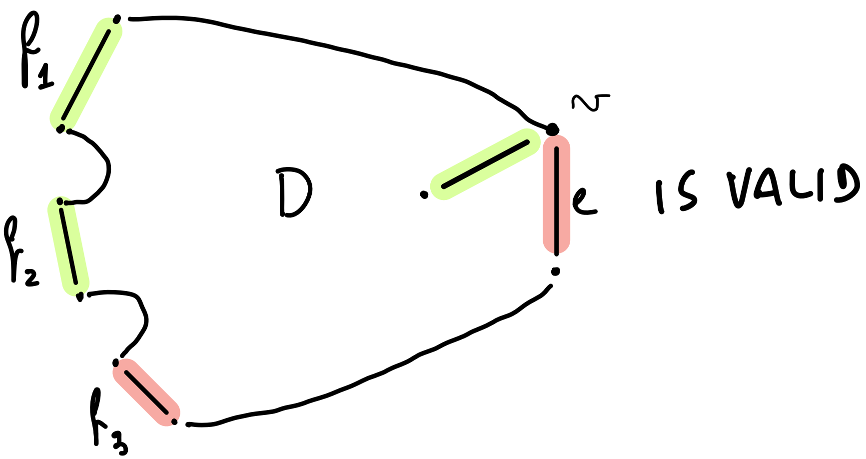
DO THEY EXIST FOR ANY F?

PROPOSITION

IF $\mathcal{F}$ IS CLOSED UNDER LOCAL FLIPPING THEN



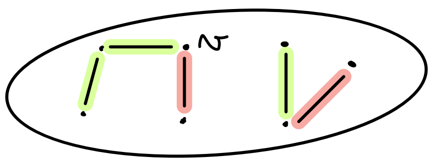
$dist(f_i, e) >> 0$



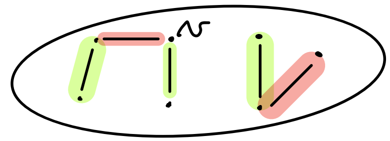
DO THEY EXIST FOR ANY $\mathcal{F}$?

PROPOSITION

IF $\mathcal{F}$ IS CLOSED UNDER LOCAL FLIPPING THEN

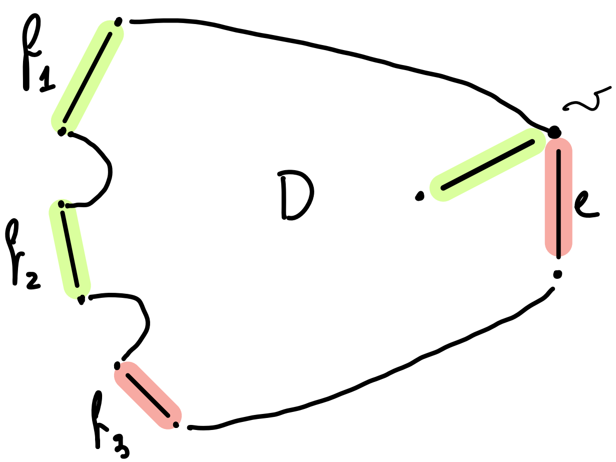


VALID $\rightsquigarrow$

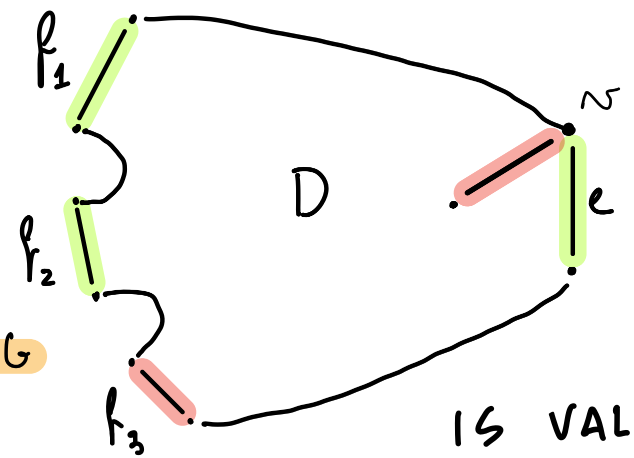


VALID

$\text{dist}(f_i, e) \gg 0$



IS VALID $\Longrightarrow$ LOCAL FLIPPING

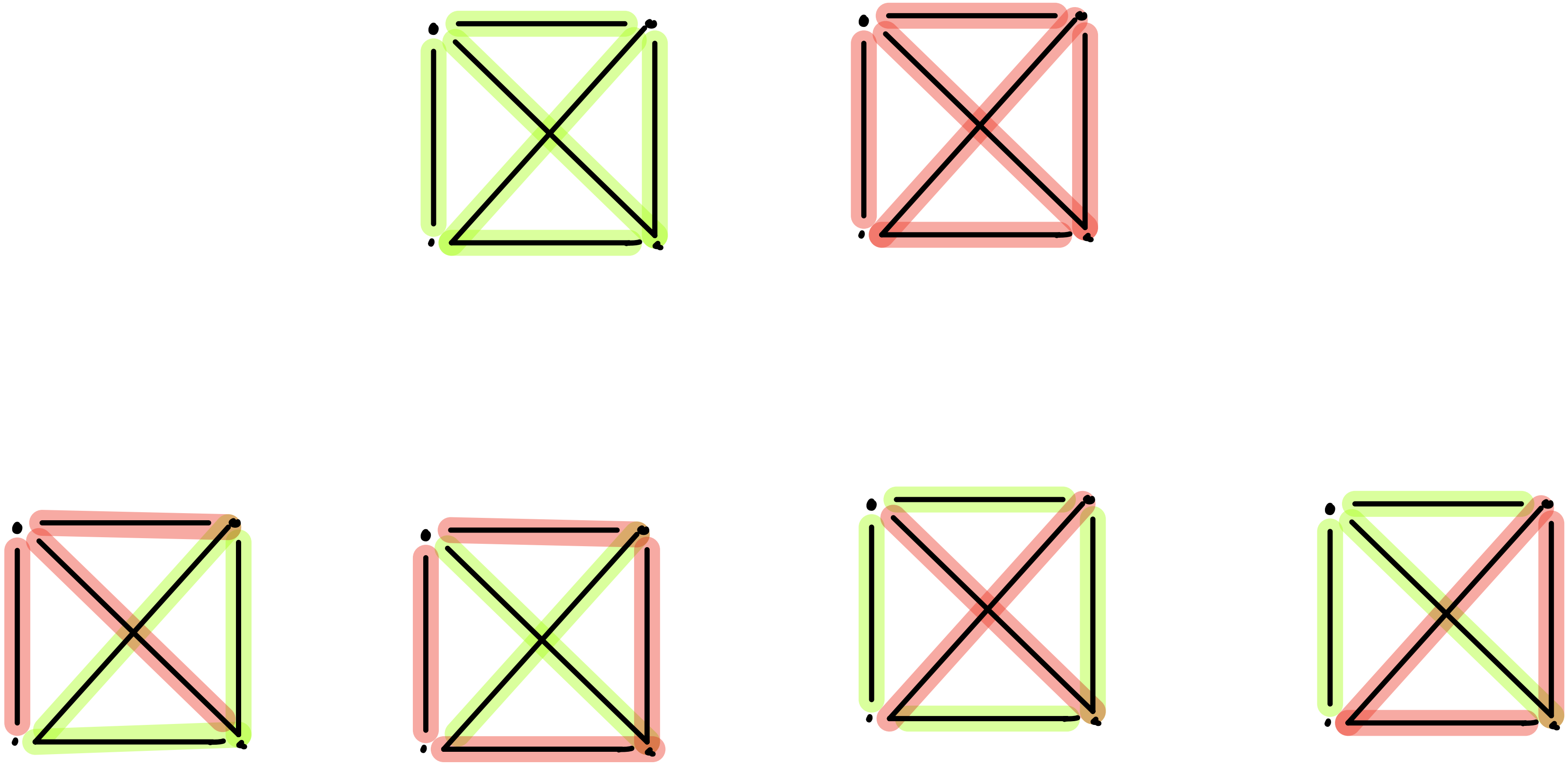


IS VALID

HENCE (D, f_i, e) CANNOT BE A COLOR DETERMINER

DO THEY EXIST FOR ANY F?

CONCRETE EXAMPLE



WHEN DO THEY EXIST?

WE CALL $\longrightarrow$ THE HOMOMORPHISM ORDER ON GRAPHS AND FOCUS ON THE FOLLOWING CHAIN

$$\cdots \; C_9 \longrightarrow C_7 \longrightarrow C_5 \longrightarrow K_3 \longrightarrow K_4 \longrightarrow K_5 \;\; \cdots$$

WHEN DO THEY EXIST?

$$\cdots C_9 \to C_7 \to C_5 \to K_3 \to K_4 \to K_5 \quad \cdots$$

THEOREM

IF $\mathcal{F}$ SATISFY THE TWO FOLLOWING REQUIREMENTS THESE GADGETS EXIST.

1. $\forall (F, \alpha) \in \mathcal{F} \quad F \in$ (CLIQUES $\cup$ ODD CYCLES)
2. THE $\to$-MAXIMUM OF $\mathcal{F}^M$ (THE MONOCHROMATIC PART) IS $\to$-SMALLER THAN THE $\to$-MINIMUM OF $\mathcal{F} \setminus \mathcal{F}^M$

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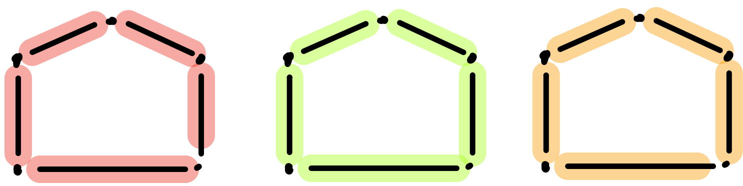
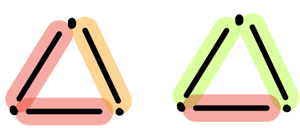
EXAMPLES

* 

* 



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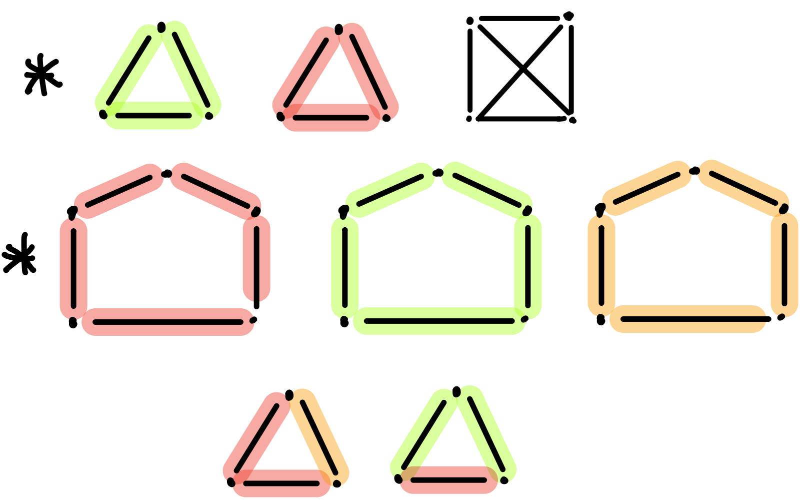
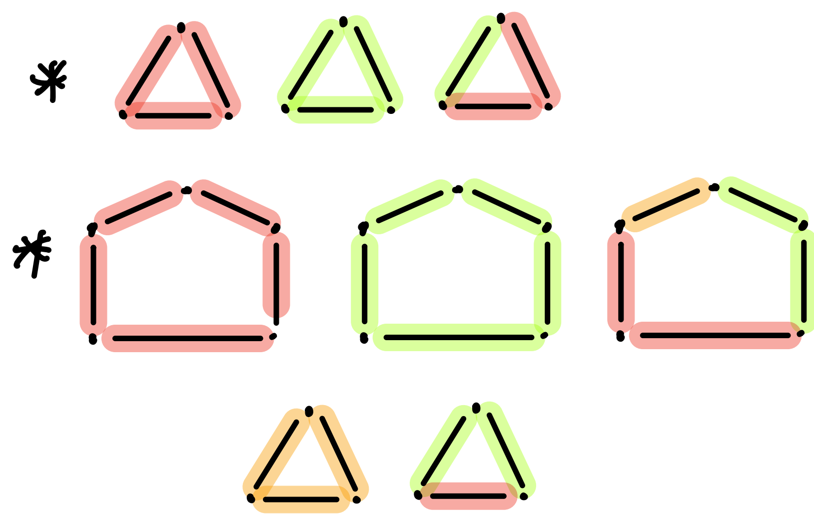


NON-EXAMPLES



THANKS!