The Fraïssé class of finite powers of simple abelian Mal'cev algebras

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Mathematics
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Let V be the vector space of countable dimension over a finite field. Does $\mathrm{GL}(V)$ have ample generics?

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Answer of experts

Should follow from WAP via CAP and Hrushovski property.

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  is **comeager**, i.e., contains the intersection of countably many dense open subsets of G^n.
- G has **ample generics** if it has generic n-tuples for each $n \in \mathbb{N}$.

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  have ample generics.
- $\mathrm{Aut}(\mathbb{Q}, \leq)$ has a generic element, but no generic pair.

A sufficient condition for ample generics

- For a Fraïssé class K, let K_p^n be the class of expansions

$$(\mathbf{A}; \phi_1, \ldots, \phi_n)$$

  where $\mathbf{A} \in K$ and $\phi_1, \ldots, \phi_n$ are isomorphisms between substructures of $\mathbf{A}$ (**partial isomorphisms**).

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Theorem (Kechris, Rosendal 2007)

Let K be a Fraïssé class with Fraïssé limit $\mathbf{M}$, let $n \geq 1$.
If K_p^n has the JEP and a cofinal subclass with AP (CAP for short), then $\mathrm{Aut}\mathbf{M}$ has generic n-tuples.

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7. Hence $\mathrm{GL}(F^{(\omega)})$ has ample generics.

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$$\mathbf{A}_1 := (F^n, x - y + z, \{ax + (1-a)y \mid a \in F^{n \times n}\}, p_r(x))$$

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Recall

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Lemma

$h\colon \mathbf{A}_i^k \to \mathbf{A}_i^\ell$ is a homomorphism iff there exist $P \in F^{\ell\times k}$ (with $P \cdot \overline{1} = \overline{1}$ in case $i = 2$) and $c \in p_r(A)^\ell$ such that

$$h(\overline{x}) = P \cdot \overline{x} + c \quad \text{for all } \overline{x} \in A^k.$$

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where 2^ω is the Cantor space, $e_1, \ldots, e_n$ is the list of all 1-element subalgebras of $\mathbf{A}$ and $x_1, \ldots, x_n \in 2^\omega$ are distinct.

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3. If **A** is non-abelian, then

   $$\mathrm{Aut}((\mathbf{A}^{2^\omega})^{x_1,\ldots,x_n}_{e_1,\ldots,e_n}) \cong N \rtimes \mathrm{Homeo}\,(2^\omega)_{x_1,\ldots,x_n}$$

   where N is the topological closure of $(\mathrm{Aut}(\mathbf{A})^{2^\omega})^{x_1,\ldots,x_n}_{1,\ldots,1}$.

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   where 2^ω is the Cantor space, $e_1, \dots, e_n$ is the list of all 1-element subalgebras of $\mathbf{A}$ and $x_1, \dots, x_n \in 2^\omega$ are distinct.
3. If $\mathbf{A}$ is non-abelian, then

   $$\mathrm{Aut}((\mathbf{A}^{2^\omega})^{x_1,\dots,x_n}_{e_1,\dots,e_n}) \cong N \rtimes \mathrm{Homeo}(2^\omega)_{x_1,\dots,x_n}$$

   where N is the topological closure of $(\mathrm{Aut}(\mathbf{A})^{2^\omega})^{x_1,\dots,x_n}_{1,\dots,1}$.
4. $\mathrm{Aut}((\mathbf{A}^{2^\omega})^{x_1,\dots,x_n}_{e_1,\dots,e_n})$ has ample generics.