Endomorphism kernel property for finite groups

Heghine Ghumashyan
(joint work with J. Guričan)

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Outline

We will talk about endomorphism kernel property for finite groups. Blyth, Fang, and Silva (2004) defined an endomorhism kernel property (EKP) for an universal algebra as follows:

Definition

An algebra A has the *endomorphism kernel property* (=EKP) if every congruence relation on A different from the universal congruence ($A \times A$) is the kernel of an endomorphism on A.

This is equivalent to the fact that every non-trivial epimorphic image of A is isomorphic to a subalgebra of A.

They proved that finite Boolean algebras, finite bounded distributive lattice has EKP if and only if it is a product of chains. They also proved a full characterisation of finite de Morgan algebras having EKP, also other classes of algebras were studied from this point of view later.

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The notion of strong endomorphism property was defined by Blyth and Silva in 2008.

Let A be a universal algebra, $f : A \to A$ be an endomorphism, $\Theta \in Con(A)$ be a congruence on A. We say that f is *compatible* with Θ if $a \equiv b \mod \Theta \Rightarrow f(a) \equiv f(b) \mod \Theta$.

Endomorphism f is *strong* (*congruence preserving*) (on A), if it is compatible with every congruence $\Theta \in Con(A)$.

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An algebra A has the *strong endomorphism kernel property* (=SEKP) if and only if every congruence relation on A different from the universal congruence ι_A is the kernel of a strong endomorphism on A.

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Notions of SEKP and EKP were considered and characterized for many algebraic structures like Ockham algebras, finite Boolean algebras, finite Brouwerian algebras, distributive lattices (bounded and also unbounded) and many others.

Most of classes of algebras considered so far were classes which have a lattice reduct. First result concerning classical structures is in the paper by J. Fang and Z.-J. Sun from 2020 - they proved that finite abelian group has SEKP if and only if it is a cyclic group.

We are going to talk about EKP for finite groups. Our main result is that every finite abelian group has EKP. We shall also discuss some results about non-abelian finite groups.

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Outline

Usual method to prove some results for EKP is by using direct product decomposition. But as congruences in (even abelian) groups are not factorable, we must be quite carefull. Nilpotency is of great help here. There is an important structural characterization of finite nilpotent groups: Let G be a finite group, $|G| = p_1^{a_1} \cdots p_k^{a_k}$, where $p_1, \ldots, p_k$ are pairwise different prime numbers. Then G is nilpotent if and only if

$$G \cong G_1 \times G_2 \times \cdots \times G_k \qquad (1)$$

where G_i is (isomorphic to) a Sylow p_i-subgroup of G for every $i \in \{1, \ldots, k\}$, it means that $|G_1| = p_1^{a_1}, \ldots, |G_k| = p_k^{a_k}$.

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We shall use the following well known theorem

Theorem

Let G be a finite nilpotent group written in this way as a product of its Sylow p_i-groups G_i,

$$G = G_1 \times G_2 \times \cdots \times G_k$$

Let H be a subgroup of G. Then there exist subgroups H_i of G_i, $i = 1, \ldots, k$, such that

$$H = H_1 \times H_2 \times \cdots \times H_k$$

Moreover, if H is a normal subgroup of G ($H \triangleleft G$), then H_i is a normal subgroup of G_i ($H_i \triangleleft G_i$) for $i = 1, \ldots, k$.

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Using this decomposition, the factor group G/H (in case when $H \triangleleft G$) can be written as a product of factor groups in the form

$$G/H \cong G_1/H_1 \times \cdots \times G_k/H_k$$

Combining previous theorems we get

Theorem

Let each of Sylow subgroups $G_1, \ldots, G_k$ of a finite nilpotent group G (written in the form (1)) have EKP. Then also G has EKP.

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As (finite) abelian groups are nilpotent, we can use this theorem and what is left is to prove that every finite abelian $p-$group has EKP.

Let us start with the following lemma which considers a special case of homomorphic images of a finite abelian $p-$group.

Lemma

Let G be a finite abelian $p-$group, K be a subgroup of G, $|K| = p$. Then the group G/K is isomorphic to a subgroup of G.

Using this it is possible to use Cauchy's theorem and a kind of induction to prove that

Theorem

Let G be a finite abelian $p-$group, $|G| = p^n$. Then for any subgroup H of G, the factor group G/H is isomorphic to a subgroup of G.

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Outline

We shall describe infinitely many finite non-abelian groups with EKP. Again, let p be a prime number.

Let G be a group, $Z(G)$ be the center of G. Using some well known properties of finite p-groups and centers of such groups, it is not difficult to prove

Theorem

Let G be a non-abelian group, $|G| = p^3$. Then there is exactly one normal subgroup of G which has p elements. Moreover, this normal subgroup is the center $Z(G)$ and

$$G/Z(G) \cong Z_p \times Z_p$$

This is the only non trivial case to be considered for non-abelian groups with p^3 elements - it means it is necessary to decide, if G has a subgroup isomorphic to $Z(G) \cong Z_p \times Z_p$ or not.

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As for each prime number p there are exactly 2 non-abelian groups with p^3 elements, there can be more straightforward way to prove what we want, but we would like to use the following beautiful statement

Theorem

Suppose that G is a $p-$group all of whose abelian subgroups are cyclic. Then G is cyclic or it is the quaternion group.

Hence, as a more–less direct consequence we have

Theorem

Let G be a non-abelian group, $|G| = p^3$, where $p > 2$, or $G \cong D_4$ (dihedral 8 element group). Then G has a non-cyclic abelian subgroup H, it means a subgroup H such that

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Therefore we know the following

Lemma

Let G be a non-abelian group, $|G| = p^3$.
1. If $p > 2$, then G has EKP.
2. If $p = 2$ and $G \cong D_4$, then G has EKP (quaternion group does not have EKP).

The previous results can be extended by multiplication using factor Z_p^k and using this and ideas about nilpotent groups, the most general result we can say at this moment is

Theorem

Let G be a finite nilpotent group written in the form (1). Let each Sylow group G_i be (isomorphic to) one of the following groups:

1. *an abelian group,*
2. $Z_{p_i}^{k_i} \times P_i$, *where* $k_i \geq 0$, $p_i > 2$ *and* P_i *is a non-abelian group of order* p_i^3,
3. $Z_2^{k_i} \times D_4$, *where* $k_i \geq 0$ *and* D_4 *is a dihedral* $8-$*element group.*

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Then G has EKP.

Remark: our results does not provide all non-abelian $p-$groups which have EKP. For example, direct computation in GAP (Groups, Algorithms and Programming) shows that there are 6 non-abelian groups of order $3^4 = 81$ which have EKP (GAP identifications returned by **IdSmallGroup()** of these groups are [81,6], [81,7], [81,8], [81,9], [81,12], [81,13]), but only 2 of them are of the form $Z_3 \times P$, where P is a non-abelian group of order 3^3. There are 4 non-abelian groups of order 81 which do not have EKP (GAP id's of these groups are [81,3], [81,4], [81,10], [81,14]).
In fact, using known classification, we made full characterization of EKP for non-abelian groups with p^4 elements, we have also fully characterized extraspecial groups which have EKP and found some interesting infinite series of special groups which enjoy EKP.

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Thank you for the attention.

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