A 10-page proof of the Dichotomy Theorem (using the Dichotomy Theorem)

Petar Marković
(joint work with Miklós Maróti and Aleksandar Prokić)

University of Novi Sad

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The Dichotomy Theorem

The Fixed-Template Constraint Satisfaction Problem

Let $\mathbb{A}$ be a fixed finite model. $CSP(\mathbb{A})$ is the decision problem which inputs a finite model $\mathbb{B}$ in the same language as $\mathbb{A}$ and accepts $\mathbb{B}$ iff there exists a homomorphism from $\mathbb{B}$ into $\mathbb{A}$.

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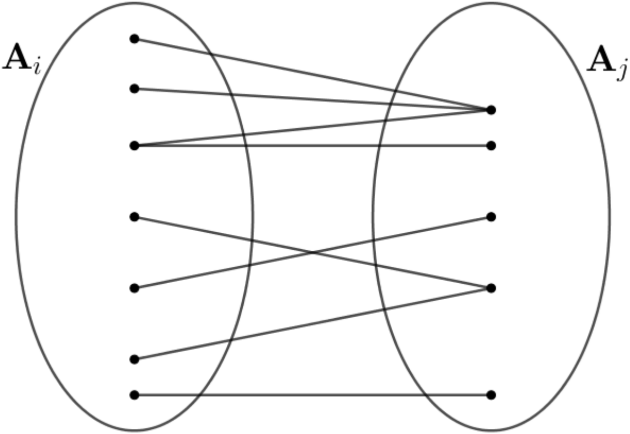
As was previously known, in all other cases $CSP(\mathbb{A})$ must be NP-complete, so a full dichotomy was thus proved.

Local consistency

In this talk, the only notion of local consistency we will use is (2,3)-minimality, which means:

Local consistency

(2-consistency): For any variables i, j, any constraint that has $\{i, j\}$ in its scope restricts to $\{i, j\}$ the same way as R_{ij}, depicted here as a graph between $\mathbf{A}_i$, the restriction to $\{i\}$, and $\mathbf{A}_j$, the restriction to $\{j\}$, and



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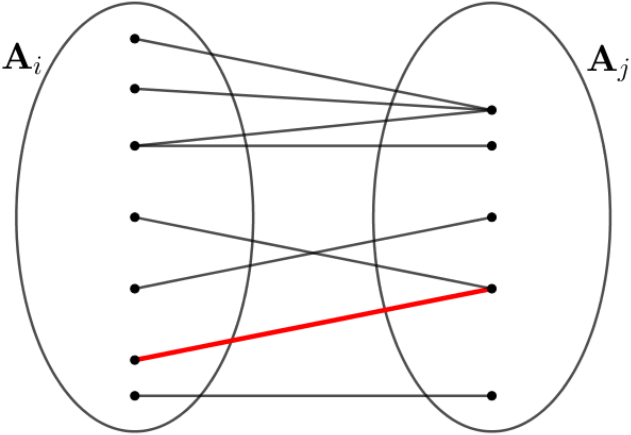
(3-density): For any edge in R_{ij},



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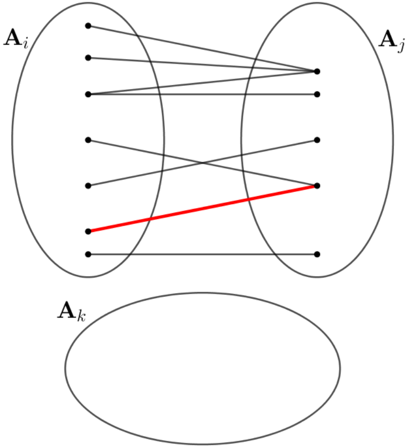
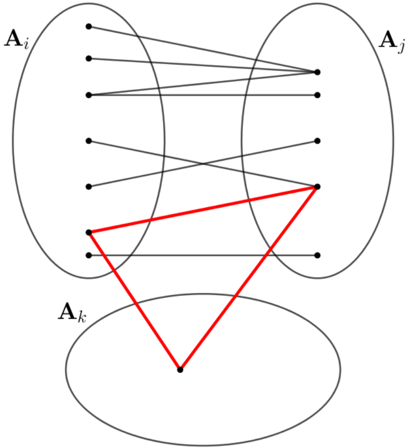


Local consistency

(3-density): For any edge in R_{ij}, and any variable k there exists a point in $\mathbf{A}_k$ such that the selected edge is in a triangle wrt. R_{ik} and R_{jk}.



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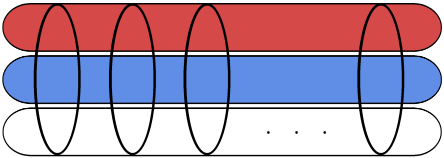

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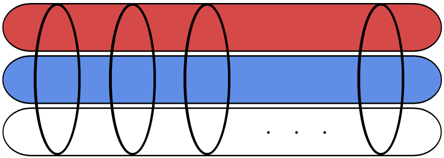

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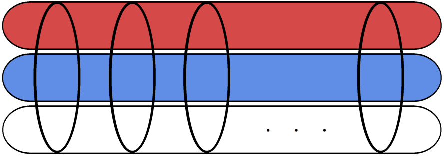

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NB: Both local consistency and structural consistency can be enforced in polytime via *universal algorithms*.

Reduction to binary ideals

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A subset B of an algebra $\mathbf{A}$ is *binary absorbing* (binary ideal) if there exists a binary term $t(x, y)$ such that $t(a, b) \in B$ whenever $|\{a, b\} \cap B| \geq 1$. If $\mathbf{A}$ is a MTA, then such subset B is a *totally absorbing subuniverse*. We will write $B \triangleleft_2 \mathbf{A}$.

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Theorem 2 (Zhuk's Reduction)

Let P be a locally and structurally consistent CSP instance compatible with a MTA. If $B \triangleleft_2 \mathbf{A}_i$, and there exists a solution of P then there exists a solution f of P such that $f(i) \in B$.

Squeezing polynomials

Let P be a (2,3)-minimal CSP instance compatible with a MTA. A tuple of polynomials $\langle p_i \in \mathrm{Pol}_1 \mathbf{A}_i : i \in V \rangle$ are *squeezing polynomials* for P if

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We are looking for a tuple of unary polynomials indexed $i \in V$ so that, for each constraint (S, R), our tuple restricts to S as a polynomial in $\mathrm{Pol}_1 \mathbf{R}$. (Note: polynomials of $\mathbf{R}$ can be represented as tuples of polynomials of $\mathbf{A}_i$, $i \in S$).

The polynomial instance

This naturally gives rise to the *polynomial instance*.

Definition 3 (The polynomial instance)

Let P be a $(2,3)$-minimal CSP instance compatible with a minimal Taylor algebra. The instance $\mathrm{Pol}\,P$ has the same variable set V as P, and for each constraint (S, R) of P, the corresponding constraint of $\mathrm{Pol}\,P$ is $(S, \mathrm{Pol}_1\mathbf{R})$.

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Note: Since P was $(2,3)$-minimal, for any constant (polynomial) $c \in A_i$, there is a constant (polynomial) in R whose ith projection is c. For any algebra $\mathbf{B}$, $\mathrm{Pol}_1 \mathbf{B}$ is the subuniverse of $\mathbf{B}^B$ generated by the identity map and the constant maps. Hence, the instance $\mathrm{Pol}\, P$ is $(2,3)$-minimal with the domain of variable i equal to $\mathrm{Pol}_1 \mathbf{A}_i$ and the digraph between variables i and j equal to $\mathrm{Pol}_1 \mathbf{R}_{ij}$.

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Any solution of $\mathrm{Pol}\, P$ is a tuple of polynomials which satisfies the first property from the previous slide. One trivial solution, when the identity map is assigned to each variable, always exists. But identity maps don't squeeze anything!

Maróti reduction: specific squeezing polynomials

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Instead, Maróti ran a singleton arc consistency algorithm together with his decomposition and was often able to eliminate elements of A_i and reduce P.

The key idea

Though $\mathrm{Pol}\, P$ is compatible with a minimal Taylor algebra (the power of the original one), it is also compatible with a binary operation, the composition. And the polynomials in $\mathrm{Pol}_1 \mathbf{A}_i$ which are not permutations binary-absorb the whole $\mathrm{Pol}_1 \mathbf{A}_i$ via the composition. Those are precisely the polynomials which DO squeeze P. We can make a new minimal Taylor reduct that $\mathrm{Pol}\, P$ is compatible with, but that also has this binary absorption.

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So we should somehow find a subset (tightening) of $\mathrm{Pol}\, P$ which is locally and structurally consistent, and contains all the identity polynomials (hence, the tightening still has the trivial solution). If at least one non-permutation remains in at least one $\mathrm{Pol}_1 \mathbf{A}_i$, we will be able to squeeze the original instance P.

The twin instance

We assume that P is a locally and structurally consistent instance compatible with a minimal Taylor algebra, and that no $\mathbf{A}_i$ has a proper binary ideal. Moreover, we assume that $M(P)$, generated using Bulatov's binary "universal meet", has no solution which squeezes the instance P. Such an instance P is called "prepared"; the methods we previously considered don't help us.

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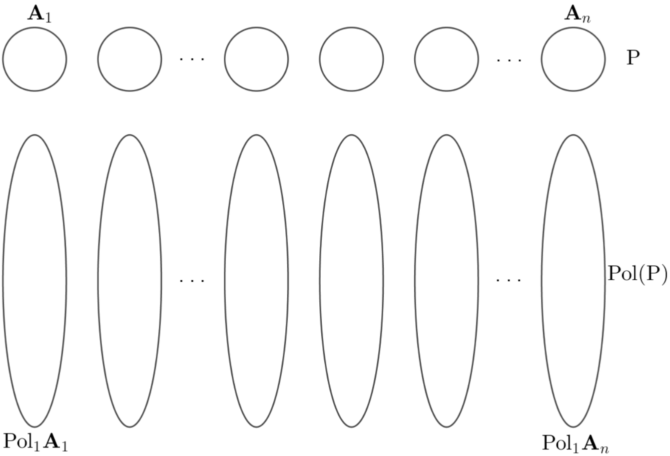
We constructed an instance $Tw(P)$, a tightening of $\mathrm{Pol}\, P$, such that

1. The instance $Tw(P)$ is compatible with a Taylor operation,
2. The instance $Tw(P)$ is locally and structurally consistent, so we denote the domain of variable $i \in V$ by $T_i \subseteq \mathrm{Pol}_1 \mathbf{A}_i$,
3. Every polynomial in every constraint relation (also in every T_i) is a twin of the identity, and idempotent (meaning $p \circ p = p$),
4. For every $i \in V$ such that $\mathbf{A}_i$ has a semilattice edge, T_i contains a polynomial which maps two endpoints of some semilattice edge into the same element, and
5. For every $i \in V$ such that $\mathbf{A}_i$ has no semilattice edges, $T_i = \{id_{A_i}\}$.

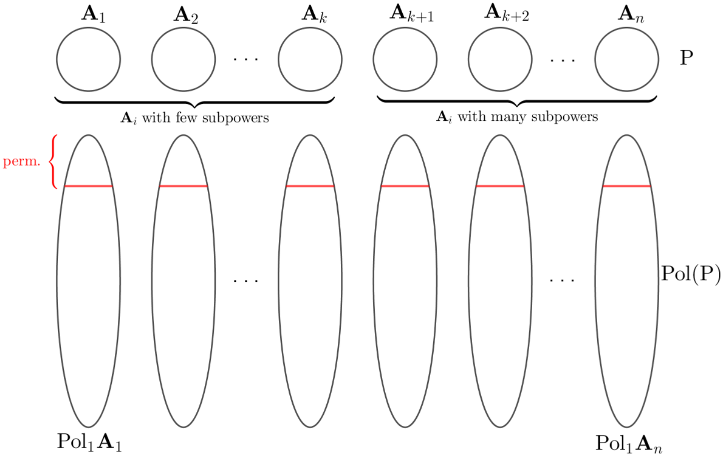
The twin instance

We start from the instances P and $\mathrm{Pol}(P)$.



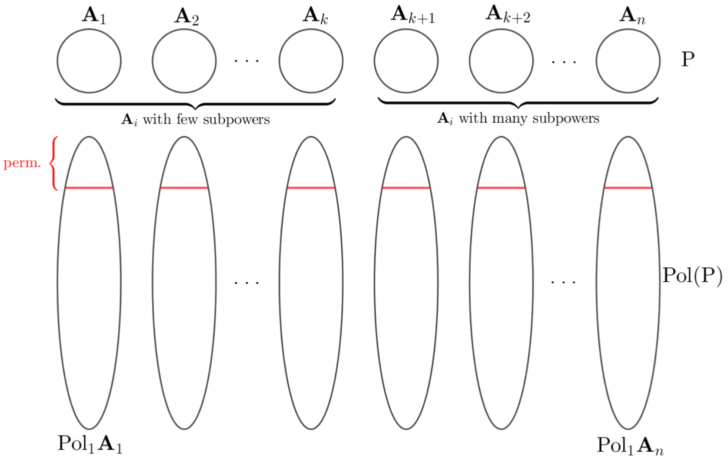
The twin instance

Let the first k have "few subpowers" (no semilattice edges), and the rest "many subpowers" (have semilattice edges).



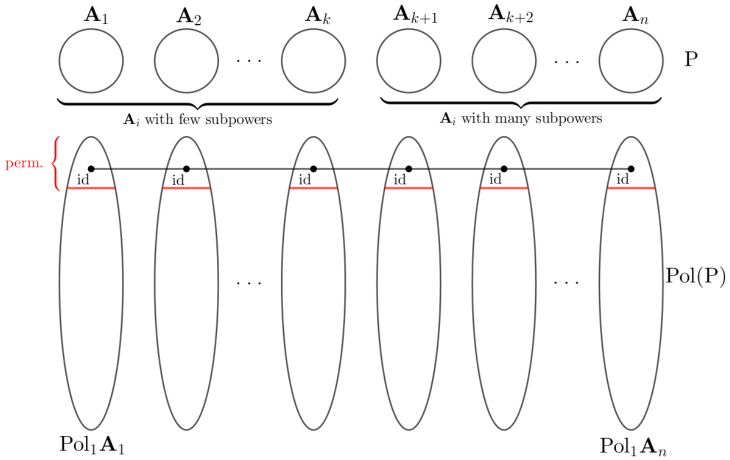
The twin instance

The non-permutation polynomials (below the red line) binary absorb all polynomials.



The twin instance

There is a solution to $\mathrm{Pol}(P)$ which always selects the identity map.



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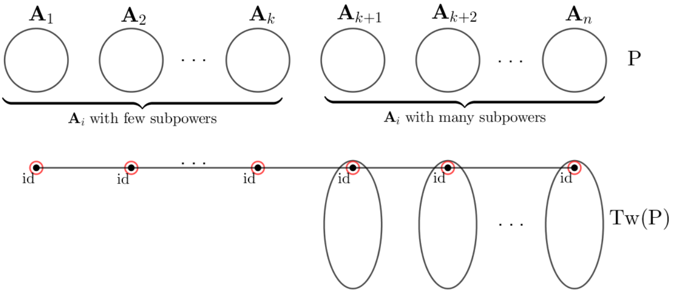
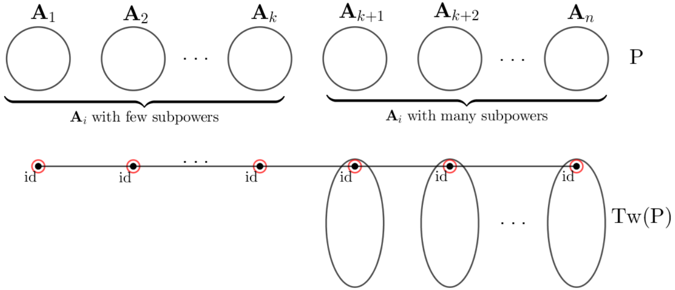
We construct $\mathrm{Tw}(P)$, a tightening of $\mathrm{Pol}(P)$, which is compatible with Taylor polymorphisms and closed under $\circ$.



The twin instance

For any $i \in V$ such that $\mathbf{A}_i$ has few subpowers, the corresponding domain of $\mathrm{Tw}(P)$ contains only one map, the identity,



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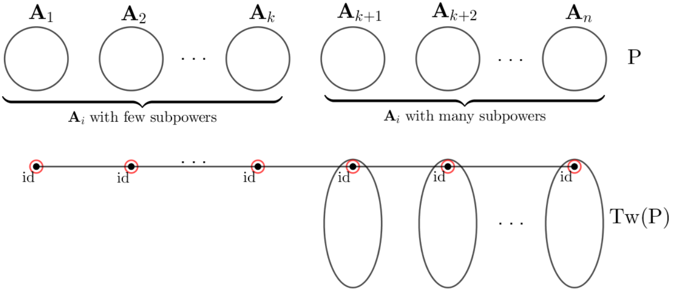
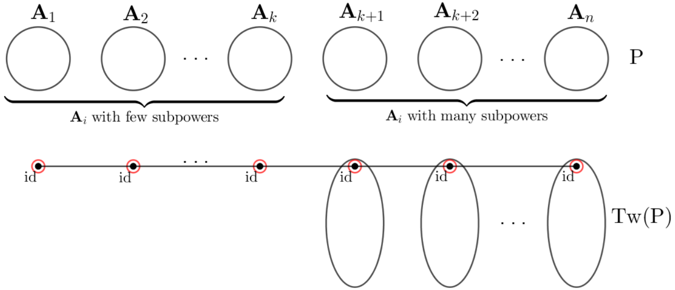
while for the remaining $i \in V$, the ith domain of $\mathrm{Tw}(P)$ contains only the identity among permutations, and must contain some non-permutations.



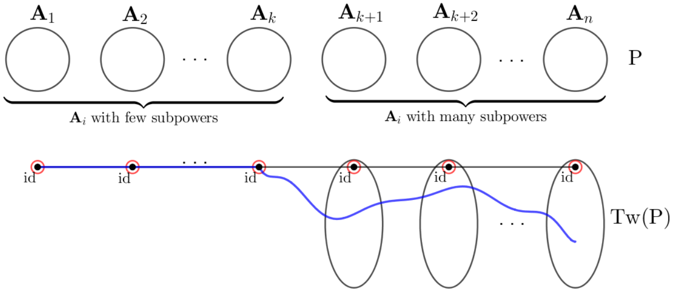
The twin instance

Always picking the identity solves $\mathrm{Tw}(P)$, also, and $\mathrm{Tw}(P)$ is constructed to be $(2,3)$-minimal and Zhuk irreducible.

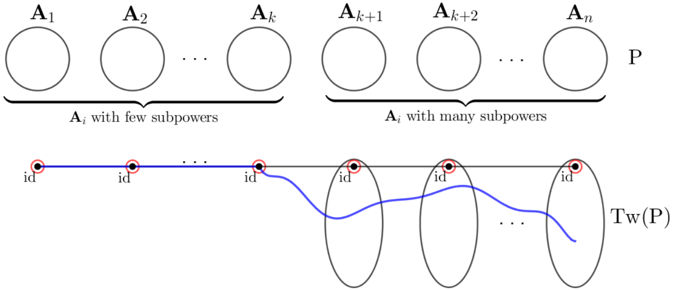


The twin instance

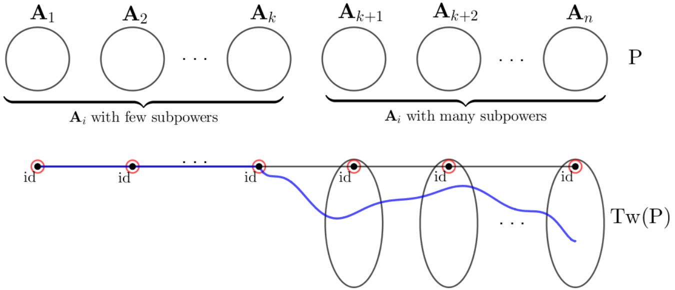
Hence, by Zhuk's Reduction Theorem, there is a solution of $\mathrm{Tw}(P)$ which selects non-permutations for all $i \in V$ for which $\mathbf{A}_i$ has many subpowers.



The solution to $\mathrm{Tw}(P)$ is also a solution to $\mathrm{Pol}(P)$, thus gives a tuple of polynomial maps which squeeze all $\mathbf{A}_i$ with many subpowers.



Finally, when no more $\mathbf{A}_i$ with many subpowers exist, the instance P can be solved with the well-known "few subpowers" algorithm.



The algorithm

So our algorithm works like this:

1 Input an instance of $CSP(\mathcal{T})$.

2 Tighten the instance to make it prepared.

3.1 If there exists $i \in V$ such that $\mathbf{A}_i$ has many subpowers, do

- 3.1.1 Construct the auxiliary instance $\mathrm{Tw}(P)$, its size is constant times bigger. Said constant is $\leq |A|^{|A|-1}$, but that's how complexity works :).
- 3.1.2 Find a solution $\mathbf{p}$ of $\mathrm{Tw}(P)$ such that, for any $i \in V$, if $\mathbf{A}_i$ has many subpowers, then $\mathbf{p}_i$ is a proper retraction.
- 3.1.3 Retract P to $\mathbf{p}(P)$. Any $\mathbf{A}_i$ with many subpowers got smaller.
- 3.1.4 Go back to Step 2.

3.2 If all $\mathbf{A}_i$ have few subpowers, solve P using the few subpowers algorithm.

Caveats and remarks

In the last slide we used Zhuk's Reduction, which is a theorem claiming the existence of a solution through binary absorbing ideals, to actually construct in Step 3.1.2 such a solution and use it as a part of our CSP-solving algorithm.

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Indeed, until someone proves Zhuk's Reduction Theorem completely independently from the Dichotomy Theorem, and even strengthens it to the effective algorithm we use, we don't have a short proof of the Dichotomy Theorem in the above slides.

What remains

To offset the constructive use of Zhuk's Reduction Theorem, it is possible to represent each $p(x) \in T_i$ as a tuple in $A_i^{A_i}$, like in Maróti's original idea. However, the number of variables increases, while the size of their domains is only guaranteed not to increase, but not to strictly decrease (as was the case in Maróti's original proof). A modification of our construction of $Tw(P)$ may fix this.

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Finally, this inspired us to look at Dichotomy algorithms from the standpoint of Descriptive Complexity. Hopefully, solving any tractable CSPs except for Mal'cev CSPs over solvable algebras can be described in the fixed-point logic. Hence, we hope that the class P within the CSPs can be described by fixed-point logic with Rank operator, or at least some generalization of Rank (from finite fields to finite Abelian groups viewed as $\mathbb{Z}$-modules).

THANK YOU FOR YOUR ATTENTION!