n rings and subrings

Abstract

We study unitary rings satisfying the identity $x^{p+k} = x^p$ for some integers p and k. We are mainly interesting in the following cases:

(1) unitary rings satisfying $x^{p+1} = x^p$. We show that these rings are of characteristic 2 (i.e. $x + x = 0$) and, moreover, they are Boolean (i.e. $x.x = x$).

(2) unitary rings satisfying $x^p = x$ for some $p \geq 3$ which are of characteristic 2. All such rings are direct products of finite fields and hence they need not be Boolean if and only if
p = m(2^kï 1) + 1 for some positive m and $k \geq 2$.

(3) unitary rings satisfying $x^{p+2} = x^p$. They need not be Boolean in a general case, however, the unitary ring satisfying the identity $x^3 = x$ is Boolean. We define a certain lattice-like structure by setting

$$x \vee y = x + y + x^p y^p \ , \ x \wedge y = x.y \ \text{and} \ x' = 1 + x \ .$$

We show that the original ring can be reconstructed from this lattice-like structure by setting

$$x + y = (x \wedge y') \vee (x' \wedge y) \quad \text{and} \quad x.y = x \wedge y \ .$$

(4) A unitary ring R of characteristic 2 satisfying $x^{p+k} = x^p$, where $k = 2^s$, need not be Boolean but the set $\{x^k ; x \in R\}$ is its unitary Boolean subring.

If R satisfies $x^{p+k} = x^p$ for k = 2^sï 1, then R is of characteristic 2 and it has a Boolean subring which is formed by all finite sums of k-powers x_1^k +é x_n^k of elements from R.