Wiener Algebra Seminar
TU Wien 2011

Maximal Centralizing Monoids and Minimal Clones

Hajime MACHIDA

(Tokyo, Japan)

For a non-empty set A, $\mathcal{O}_A^{(n)}$ denotes the set of n-variable functions defined on A and $\mathcal{O}_A$ denotes the union of $\mathcal{O}_A^{(n)}$ for all $n > 0$. For a subset F of $\mathcal{O}_A$ the *centralizer* F^* of F is the set of functions in $\mathcal{O}_A$ which commute with all functions in F.

A submonoid M of $\mathcal{O}_A^{(1)}$ is called a *centralizing monoid* if $M^{**} \cap \mathcal{O}_A^{(1)} = M$ holds. It is equivalent to saying that M is a unary part of some centralizer, i.e., $M = F^* \cap \mathcal{O}_A^{(1)}$ for some $F \subseteq \mathcal{O}_A$. If $M = F^* \cap \mathcal{O}_A^{(1)}$ is satisfied for a centralizing monoid M and a subset F of $\mathcal{O}_A$, we call F a *witness* of M. A maximal centralizing monoid has a singleton witness.

On the three-element set $A = \{0, 1, 2\}$, we explicitly determine all maximal centralizing monoids. There are 10 maximal centralizing monoids. Surprisingly, all maximal centralizing monoids have some specific *minimal functions* as their witnesses. More precisely, 3 of maximal centralizing monoids have constant functions, which are minimal functions, as their witnesses and 7 of maximal centralizing monoids have majority minimal functions as their witnesses.

Furthermore, for any $k\,(> 2)$ element set A, it is proved that a centralizing monoid having a constant function as its witness is always a maximal centralizing monoid.