Analysis 2 for Computer Science

Exercise class 9

Problem 1) Solve the following partial differential equation. $u_{xx} + u_x + x + y = 1$

Problem 2) Solve the following partial differential equation. $xyu_x + u_y = xy\cos(x)$

Problem 3) We consider the following partial differential equation:

$$xu_x + 2yu_y = 0.$$

(*a*) Determine the general solution of this differential equation.

(*b*) Determine the solution which also satisfies:

$$u\left(x, \frac{1}{x}\right) = x.$$

Problem 4) A function $u(x, y)$ is called *homogeneous* of degree n, if

$$u(\lambda x, \lambda y) = \lambda^n u(x, y)$$

holds for all $\lambda > 0$ and x, y. Differentiate this equation with respect to λ to show: if u is furthermore a continuously differentiable function, then it satisfies the following linear partial differential equation of order 1:

$$xu_x + yu_y = nu.$$

What is the general solution of this differential equation?

Problem 5) Solve the initial value problem

$$u_t + u^2 u_x = 0, \qquad u(x, 0) = x.$$

Hint: Show first, that u satisfies $u = f(x - tg(u))$.

Problem 6) Classify the following partial differentail equations of order 2 (hyperbolic, parabolic or elliptic):

(*a*) $u_{xx} + 2u_{xy} + u_{yy} + u_x + u_y = 0$,

(*b*) $u_{xx} + 2u_{xy} + 5u_{yy} + u_x + u = 0$,

(*c*) $3u_{xx} - 8u_{xy} + 4u_{yy} - u = 0$,

(*d*) $u_{xy} + xyu_{yy} + u_y = 0$,

(*e*) $u_{xx} + yu_{yy} + \frac{1}{2}u_y = 0$.