AofA 2015
Abstracts of the talks

Invited talks

Universality of Scaling Limits of Critical Random Graphs

Nicolas Broutin, Inria Paris-Rocquencourt

Over the last few years a wide array of random graph models have been postulated to understand properties of empirically observed networks. Most of these models come with a parameter t (usually related to edge density) and a (model dependent) critical time t_c which specifies when a giant component emerges. There is evidence to support that for a wide class of models, under moment conditions, the nature of this emergence is universal and looks like the classical Erdős-Rényi random graph, in that the extent of the range of the parameter in which phase transition occurs, but also of the sizes and structures of the large connected components within this range (when equipped with the graph distance) converge to random fractals related to Aldous' celebrated continuum random tree.

Our aim is to present and discuss a general program for proving such results. The results apply to a number of fundamental random graph models including the configuration model, inhomogeneous random graphs modulated via a finite kernel and bounded size rules.

Analysis of `Quicksort` with More than One Pivot

Martin Dietzfelbinger, Technische Universität Ilmenau
Joint work with Martin Aumüller

Dual-pivot quicksort refers to variants of classical quicksort where in the partitioning step two pivots are used to split the input into three segments. This can be done in different ways, giving rise to different algorithms. Starting from 2009, a dual-pivot algorithm due to Yaroslavskiy received much attention, because it replaced the well-engineered quicksort algorithm in Oracle's Java 7 runtime library. Nebel and Wild (ESA 2012) analyzed this algorithm and showed that on average it uses $1.9n \ln n + O(n)$ comparisons to sort an input of size n, beating standard quicksort, which uses $2n \ln n + O(n)$ comparisons. We consider a model that captures all dual-pivot algorithms, give a unified analysis, and identify new dual-pivot algorithms that minimize the average number of key comparisons among all possible algorithms up to a linear term. This minimum is $1.8n \ln n + O(n)$. We also consider the generalized situation when there are $k > 2$ pivots. Kushagra *et al.* (ALENEX 2014) described one such algorithm. We can identify an abstract optimal algorithm for all k; the exact value of the optimal expected number of comparisons can so far be determined only for $k = 3$. (It is $\frac{133}{72} n \ln n \approx 1.705 n \ln n$.) We also comment on variants that choose pivots from a sample and other parameters like the number of data movements or cache faults that influence the empirical performance of a quicksort algorithm.

A Line-breaking Construction of the Stable Trees

Christina Goldschmidt, University of Oxford

Consider a Galton-Watson branching process with a critical offspring distribution, conditioned to have total progeny n. The family trees of such processes constitute a natural collection of models for random trees, which includes various standard combinatorial trees such as uniform random labelled trees and uniform binary planar trees. Since 1990, a beautiful theory of scaling limits for these objects has been developed. It turns out that there is a good way to rescale distances in the tree so that, in the limit as n tends to infinity, one obtains a compact limit object. The family of possible limiting objects, which are essentially "tree-like" path metric spaces, is now known as the stable trees. I will give a survey of some of this theory, and then talk about joint work with Bénédicte Haas (Paris-Dauphine), in which we give a new almost sure construction of the stable trees, via a surprisingly elementary line-breaking procedure.

The Surprising Power of Belief Propagation

Elchanan Mossel, University of California Berkeley

Belief Propagation is a very efficient and popular algorithm for inference. While it is long known that the algorithm is exact for trees, its applicability for non-tree graphs have been intensively studied in coding theory, statistical physics and theoretical computer science for the last few decades. I will present the algorithm and some old and newer theoretical guarantees for its performance.

Analytic Combinatorics in Biology

Markus Nebel, Technische Universität Kaiserslautern

At the latest with the computerization of many branches of their research, discrete models became of increased importance to biologists. With the application of those models various quantitative questions were posed, most of which could be answered by methods from analytic combinatorics. Often, a special feature in this domain is the use of formal languages instead of directly applying the symbolic method. In this talk we will discuss some representative examples showing the elegance of the methods and also its enhancements in the context of biology.

Examples of Two-parameter Recurrences and their Treatments

Alois Panholzer, Technische Universität Wien

Recurrences occur "almost everywhere" in the analysis of algorithms and random combinatorial structures. Whereas the (asymptotic) analysis of one-parameter recurrences is in many cases well-established by standard techniques or general theorems (as the "master theorem"), studies of two- (and more) parameter recurrences often remain "tricky". We give here some recent examples of, as we find, interesting problems where during their treatments such recurrences pop up. In particular, we will comment on a detailed analysis of the so-called $(1+1)$-evolutionary algorithm (together with Hsien-Kuei Hwang, Nicolas Rolin, Tsung-Hsi Tsai and Wei-Mei Chen), on generalizations of parking functions for trees and mappings (together with Marie-Louise Bruner), and on the characterization of certain limiting distributions as Mixed Poisson distributions (together with Markus Kuba).

Symbolic Summation for Combinatorial and Related Problems

Carsten Schneider, RISC Linz

Symbolic summation can be considered as one of the key technologies to solve combinatorial problems. In this talk I will work out the difference ring approach that supports the user to simplify multiple sums to expressions in terms of indefinite nested sums and products. In particular, I will illustrate how the summation algorithms of telescoping, creative telescoping and recurrence finding can be used to tackle combinatorial and related problems.

The Phase Transition in Achlioptas Processes

Lutz Warnke, University of Cambridge
Joint work with Oliver Riordan

In the Erdős-Rényi random graph process, starting from an empty graph, in each step a new random edge is added to the evolving graph. One of its most interesting features is the *percolation phase transition*: as the ratio of the number of edges to vertices increases past a certain critical density, the global structure changes radically, from only small components to a single giant component plus small ones.

In this talk we consider Achlioptas processes, which have become a key example for random graph processes with dependencies between the edges. Starting from an empty graph these proceed as follows: in each step two potential edges are chosen uniformly at random, and using some rule one of them is selected and added to the evolving graph. We discuss why, for a large class of rules, the percolation phase transition is qualitatively comparable to the classical Erdős-Rényi process.

Contributed talks

Most Trees Are Short and Fat

Louigi Addario-Berry, McGill University

Let T be any critical or subcritical Galton-Watson tree. Write $ht(T)$ for the height of T (the greatest distance of any node from the root) and $wid(T)$ for the width of T (the greatest number of nodes at any level). We study the relation between $ht(T)$ and $wit(T)$.

In the case when the offspring distribution $p = (p_i, i \geq 0)$ has finite variance, both $ht(T)$ and $wid(T)$ are typically of order $vol(T)^{1/2}$, and have sub-Gaussian upper tails (A-B, Devroye and Janson, 2013). Heuristically, as the tail of the offspring distribution becomes heavier, the tree T becomes "shorter and bushier". We prove a theorem which can be viewed as justifying this heuristic. We also state a combinatorial conjecture which, if true, would yield a substantial strengthening of our main result: informally, the conjecture is that binary trees are the tallest.

Strong Randomness Properties of (Hyper-)Graphs Generated by Simple Hash Functions

Martin Aumüller, Technische Universität Ilmenau
Joint work with Martin Dietzfelbinger and Philipp Woelfel

We study the randomness properties of graphs and hypergraphs generated by a set of keys and hash functions. We present a simple construction for a class of hash functions that have small constant evaluation time and sublinear space consumption. Graphs built with these hash functions behave fully randomly in many structural aspects. We will apply this property to the study of randomized algorithms and show that our class of hash functions exhibits very good behaviour in cuckoo hashing with a stash and randomized load balancing. An interesting point in the analysis is that it only relies on random graph theory without having to worry for details of the hash function.

Some Unusual Asymptotics for a Variant of Insertion Sort

Cyril Banderier, Université Paris 13
Joint work with Jean-Luc Baril and Céline Moreira Dos Santos

In the Art of Computer Programming (Vol 3. Sorting and Searching), D.E. Knuth made a detailed analysis of variants of insertion sort algorithms; some of them rely on the *insertion-sorting operator* defined by

$$\sigma_1\sigma_2\ldots\sigma_n \longrightarrow \sigma_1\ldots\sigma_j\sigma_i\sigma_{j+1}\ldots\sigma_{i-1}\sigma_{i+1}\ldots\sigma_n.$$

We enumerate the number of permutations reachable from the identity after a given number of such transformations: we give the bivariate exponential generating function, its asymptotics and the corresponding limit law. It is a nice surprise that it involves an unusual algebraic exponent, and some unusual closed-form constants. (From an analytic combinatorics point of view, all of this is typical of a movable exponent singularity, or of parametrized linear differential equations.)

Large Scaling Laws for Maximum Coloring of Sparse Random Geometric Graphs

Milan Bradonjić, Bell Labs
Joint work with Sem Borst

We study maximum coloring of sparse random geometric graphs, in an arbitrary but constant dimension, with a constant number of colors. We show laws of large numbers as well as central limit theorem type results for the maximum number of vertices that can be properly colored. Since this functional is neither scale-invariant nor smooth, we design tools that with the main method of sub-additivity allow us to show the weak and strong laws. Additionally, by proving the Lindeberg conditions, we show the normal limiting distribution for this functional.

Metrical Versions of the Two Distances Theorem

Eda Cesaratto, Universidad Nacional de General Sarmiento
Joint work with Brigitte Vallée

We perform a probabilistic study of five parameters — two distances, covered space, discrepancy and Arnold measure — which describe the celebrated Kronecker sequence $\mathcal{K}(\alpha)$ of the fractional parts of the multiples of a real α. Each parameter can be viewed as a measure of randomness of the sequence $\mathcal{K}(\alpha)$, and is useful to study the pseudo-randomness of the sequence $\mathcal{K}(\alpha)$ for a random input α. We wish to answer the question: Is a random Kronecker sequence pseudo-random (with respect to each randomness measure)? We consider two main cases: the case where the "input" α is a random real, and the case when α is a random rational, and we exhibit strong similarities between the two situations. It is already known that the size of the quotients in the continued fraction expansion of α plays an important role. This is why we also focus to the "constrained" case where all the quotients are bounded by a constant M, and consider the transition between the constrained case $M < \infty$ and the unconstrained case $M = \infty$.

On quick and energy-efficient leader election protocols

Jacek Cichoń, Politechnika Wrocławska

TBA

Graphs with Degree Constraints

Élie de Panafieu, RISC Linz
Joint work with Lander Ramos

Several classic families of graphs are characterized by constraints on the degrees of their vertices. Eulerian graphs, regular graphs, and graphs with a lower bound on the degrees are the most common examples. Furthermore, using the generating function of those graphs and the symbolic method, other families can be described. For example, the graphs that contain no trees are the graphs of minimum degree at least 2, where each vertex is replaced with a rooted tree.

We derive the generating functions and the asymptotics of graphs where all vertices have their degrees in a given set. Those results are then extended to the case where each vertex has its own set of allowed degrees, under some simple conditions on those sets. The previous related results were the enumeration of graphs with a given degree sequence (Bender Canfield 1978), and of graphs with a minimum degree bound (Pittel Wormald 2003).

Our proofs relies on analytic combinatorics. The generating function is first obtained for multigraphs, where loops and multiple edges are allowed. An inclusion-exclusion technique is then applied to remove loops and multiple edges.

Limiting Distributions for `QuickVal` Residuals, for Key Comparisons and Symbol Comparisons: Scale Mixtures of Gaussian Laws

Jim Fill, Johns Hopkins University
Joint work with Jason Matterer

Let $X_1, X_2, \dots$ be i.i.d. keys drawn from a probabilistic source; each key is an infinite string of symbols from the finite r-ary alphabet $\{0, 1, \dots, r-1\}$ or the countably infinite alphabet $\{0, 1, \dots\}$. Keys are ordered by lexicographic order.

Fix $\alpha \in [0, 1]$. The algorithm $\texttt{QuickVal}(n)$ uses `QuickSelect` to search among $X_1, \dots, X_n$ for the (*population*) α-quantile of the probabilistic source; under quite mild conditions on the source, the search is almost surely unsuccessful. Let S_n denote the cost of this search, obtained by summing the costs of the individual key comparisons used by $\texttt{QuickVal}(n)$. Under mild tameness assumptions on the source and on the cost (for example, unit cost or symbol-comparison cost) of comparing two keys, Fill and Nakama (2013) proved that S_n/n has a limit V, both almost surely and in L^p for certain values of p, and identified V.

We consider the *residual* $R_n := \sqrt{n}\left(\frac{S_n}{n} - V\right)$ and prove, again under mild tameness assumptions, that R_n converges in distribution to a scale mixture of Gaussian laws. Of greater interest is the corresponding residual for the cost of $\texttt{QuickQuant}(n)$, which uses `QuickSelect` to search (successfully) for the *sample* α-quantile among $X_1, \dots, X_n$; study of that residual is still in progress. The algorithm $\texttt{QuickMin}(n)$ searches for the smallest of the n keys; since its operation is the same as for $\texttt{QuickVal}(n)$ with $\alpha = 0$, our residual limit-law result applies.

The Node Profile of Symmetric Digital Search Trees

Michael Fuchs, National Chiao Tung University
Joint work with Michael Drmota, Hsien-Kuei Hwang, Ralph Neininger

The node profile of random digital trees has been studied at least from the mid-1970s due to its connections with successful and unsuccessful search. However, very little was known beyond the mean prior to a 2009 paper of Hwang, Nicodème, Park and Szpankowski who gave a complete study of the profile of tries. The main contribution of their paper was the analysis of the asymmetric case which revealed many interesting phenomena and was highly non-trivial. The symmetric case, on the other hand, turned out to be much easier due to explicit and easy expressions for Poisson mean and variance. In this talk, we will show that similar results as for tries also hold for the profile of symmetric digital search trees whose analysis is again non-trivial. We will explain the main new obstacles which arise and how we resolve them.

B-trees, and Pólya Urns

Danièle Gardy, Université de Versailles
Joint work with Dai-Hai Ton-That, Brigitte Chauvin and Nicolas Pouyanne

Although B-trees have been widely used for half a century, their performances are still not fully understood, mostly due to their balanced nature and to the complexities it induces. After presenting some enumeration results on B-trees, one of which leads to an unexpected — at least for the authors — connection with Hanoi towers, we consider how Pólya urns may be used to model nodes at the lowest level, aka the “fringe”. We show the existence of a limiting distribution for this fringe, which exhibits a phase transition when the B-tree parameter (governing the number of keys allowed in any one node) grows. This is the transition from so-called “small” urns, and a limiting gaussian distribution, to “large” urns, for which the limiting distribution, which is not gaussian, does not seem to be already known.

Analysis of the Natural Counting of λ-terms

Zbigniew Gołębiewski, Technische Universität Wien

Grygiel and Lescanne introduced a new notion of size of a λ-term (so called "natural counting") based on the encoding λ-terms with use of De Bruijn indices. We show that the number of λ-terms of size n in this model is $Cn^{-3/2}(3.38298\ldots)^n$ as $n \to \infty$. Moreover we give the lower and upper bounds on the constant C.

Asymptotics of the Coefficients of Bivariate Analytic Functions with Algebraic Singularities

Torin Greenwood, University of Pennsylvania

Flajolet and Odlyzko (1990) derived asymptotic formulae for the coefficients of a class of univariate generating functions with algebraic singularities. These results have been extended to classes of multivariate generating functions by Gao and Richmond (1992) and Hwang (1996, 1998), in both cases by reducing the multivariate case to the univariate case. Pemantle and Wilson (2013) outlined new multivariate analytic techniques and used them to analyze the coefficients of rational generating functions. In this talk, we use these multivariate analytic techniques to find asymptotic formulae for the coefficients of a broad class of bivariate generating functions with algebraic singularities.

A Functional Central Limit Theorem for Branching Random Walks, with Applications to `Quicksort` Asymptotics

Rudolf Grübel, Leibniz Universität Hannover
Joint work with Zakhar Kabluchko

Let K_n be the number of comparisons needed by `Quicksort` to sort the first n values from a sequence $(U_i)_{i\in\mathbb{N}}$ of independent random variables. We assume that the U_i's are all uniformly distributed on the unit interval and that the respective first element of a list serves as the pivot. Régnier showed that a suitable rescaling of K_n leads to an L^2-bounded martingale, implying that, as $n \to \infty$,

$$Z_n := \frac{K_n - EK_n}{n+1} \to Z_\infty \text{ almost surely} \tag{1}$$

with some limit random variable Z_∞. Recently, Neininger obtained an associated central limit theorem,

$$W_n := \sqrt{\frac{n}{2\log n}}(Z_\infty - Z_n) \to W_\infty \text{ in distribution,} \tag{2}$$

where W_∞ has the standard normal distribution.

We use the well-established link to branching random walks to obtain a stronger result: With $(\mathcal{G}_n)_{n\in\mathbb{N}}$ the martingale filtration we obtain almost sure convergence of the conditional distributions $\mathcal{L}[W_n|\mathcal{G}_n]$ with respect to the topology of weak convergence. The proof is based on a functional version of the central limit theorem for the Biggins martingale, a result that we believe to be of independent interest.

Analysis of Bidirectional Ballot Sequences and Random Walks Ending in their Maximum

Clemens Heuberger, Alpen-Adria-Universität Klagenfurt
Joint work with Benjamin Hackl, Helmut Prodinger and Stephan Wagner

In a recent paper studying "more sums than differences (MSTD)" sets, Zhao investigated "bidirectional ballot sequences" and conjectured an asymptotic formula for their number. These correspond to non-negative lattice paths ending at their maximum height, which will be called admissible paths.

We show that the probability for a lattice path to be admissible is related to the Chebyshev polynomials of the first or second kind, depending on whether the lattice path is defined with a reflective barrier or not. Parameters like the number of admissible paths with given length or the expected height are analyzed asymptotically. This allows to prove Zhao's conjecture.

Using Pólya Urns to Prove Normal Limit Laws for Protected Nodes in m-ary Search Trees

Cecilia Holmgren, Stockholms Universitet
Joint work with Svante Janson

In this talk we will show how to apply generalised Pólya urns to estimate the number of protected nodes in m-ary search trees.

We show that the number of two-protected nodes (the nodes that are neither leaves nor parents of leaves) in a random ternary search tree is asy mptotically normal. The methods apply to m-ary search trees with larger m as well, although the size of the matrices used in the calculations grow rapidly with m. We in fact show that the number of two-protected nodes is asymptotically normal for all $m \leq 26$, although the asymptotic variances are in general hard to calculate.

The one-protected nodes, and their complement, i.e., the leaves, are easier to analyze. By using a simpler Pólya urn (that is similar to the one that has earlier been used to study the total number of nodes in m-ary search trees), we prove normal limit laws for the number of one-protected nodes and the number of leaves for all $m \leq 26$.

Generation of Random Permutations: Algorithms and Analysis

Hsien-Kuei Hwang, Academia Sinica Taiwan: **Part (I)**

Axel Bacher, Université Paris 13: **Part (II)**

We survey a few simple algorithms in the literature (CS & Stat) for generating random permutations, and give a detailed analysis of them. New algorithms whose efficiency outperform all known ones are also presented. This talk is based on joint work by Olivier Bodini and Tsung-Hsi Tsai.

Edgeworth Expansions for Branching Random Walks and Random Trees

Zakhar Kabluchko, Universität Münster
Joint work with Rudolf Grübel

Consider a branching random walk on $\mathbb{Z}$. Denote by $L_n(k)$ the number of particles located at site k at time n. We will derive an Edgeworth expansion for the so-called profile $k \mapsto L_n(k)$ as $n \to \infty$. This expansion can be used to derive limit theorems for various functionals of the profile such as the local occupation numbers, the mode and the height. Similar results can be obtained for random trees such as binary search trees or random recursive trees.

Characterisation of Symmetries of Unlabelled Triangulations

Mihyun Kang, Technische Universität Graz

Random planar graphs have attained considerable attention in the recent years. Like for the Erdős and Rényi random graph, most results are for labelled cases. For unlabelled cases, not even their asymptotic number is known. Triangulations are the most basic class of planar graphs. In this talk we discuss a full characterisation of unlabelled triangulations and derive a constructive decomposition of them. This will enable us to deduce a complete enumerative description of unlabelled cubic planar graphs.

Analysis of Parameters of Multi-base Representations of an Integer

Daniel Krenn, Technische Universität Graz
Joint work with Dimbinaina Ralaivaosaona and Stephan Wagner

In a multi-base representation of an integer (in contrast to, for example, the binary or decimal representation) the base (or radix) is replaced by products of powers of single bases. To be more precise, we consider integers written as a sum of

- integers 1, 2, ..., d acting as a digit set times
- products of powers of pairwise coprime, positive integers $p_1^{\alpha_1} p_2^{\alpha_2} \dots p_m^{\alpha_m}$ acting as a base.

The resulting numeral system is usually redundant, which means that each integer can have many different digit expansions.

The motivation studying such expansions comes from cryptography. For fast arithmetic (e.g., algorithms for exponentiation in a finite group or the scalar multiplication on elliptic curves), a "good" choice of the numeral system is an important aspect.

In this talk, we will discuss several asymptotic properties of these representations of integers. We start with the following question: What is the number of multi-base representations of a positive integer? We provide a general asymptotic formula for this number. We then focus on the analysis of various parameters of multi-base expansions, as for example the sum of digits and the Hamming weight of a random representation of a fixed integer. Asymptotically, these follow Gaussian distributions.

The proofs of these results are based on a saddle-point analysis of the associated generating functions, but several technical details have to be taken into account. To deal with the various harmonic sums that occur, we apply the classical Mellin transform technique.

Combinatorial Characterization of Transducers with Bounded Variance

Sara Kropf, Alpen-Adria-Universität Klagenfurt

In many contexts, a necessary condition for an asymptotic limit theorem is the variability condition, that is the variance has to be unbounded. We give a combinatorial characterization of transducers whose output sum does not satisfy this variability condition: The output of each cycle of the transducer has to be proportional to its length.

This characterization allows to prove a central limit theorem for a sequence for which there exists a transducer that it is too large or not explicitly given. For example, we can prove that the variability condition is satisfied for the optimal Hamming weight of many τ-adic digit expansions for an algebraic number τ.

Limit Distributions for Urn Models with Multiple Drawings

Markus Kuba, FH Technikum Wien

Consider a balanced two-color Pólya-Eggenberger urn model with multiple drawings and sample size $m \in \mathbb{N}$. We extend the well known trichotomy of limit laws — small urns with a Gaussian limit, large urns with a non-Gaussian limit, and triangular urns — from sample size $m = 1$ to arbitrary $m \geq 1$ for the class of linear affine urns. We also present explicit results for the expected value and second moment, and provide asymptotic expansions of the variance for this class (joint work with Hosam Mahmoud). Finally, a characterization of r-color linear affine urn models, $r \geq 2$, with multiple drawings is briefly discussed, and it is indicated how analytic combinatorics can (re-)derive and extend these results (joint work with Basile Morcrette).

Issues in Random Networks: The Apollonian Network as a Case Study

Hosam Mahmoud, George Washington University

We briefly review a number of random networks in recent areas of interest of the speaker, and discuss the issues that arise. We present the Apollonian network as a case study. We study the distribution of the degrees of vertices as they age in the evolutionary process. Asymptotically, the (suitably-scaled) degree of a node with a fixed label has a Mittag-Leffler-like limit distribution. The degrees of nodes of later ages have different asymptotic distributions, influenced by the time of their appearance. The very late arrivals have a degenerate distribution. The result is obtained via triangular Pólya urns. Also, via the Bagchi-Pal urn, we show that the number of terminal nodes asymptotically follows a Gaussian law. We prove that the total weight of the network asymptotically follows a Gaussian law, obtained via martingale methods. Similar results carry over to the sister structure of the k-trees.

Condensation and Symmetry-breaking in the Zero-range Process with Weak Site Disorder

Cécile Mailler, University of Bath

The *zero-range process* (ZRP) is described as follows: n sites contain respectively $(Q_1, \ldots, Q_n)$ particles, where the $(Q_1, \ldots, Q_n)$ are i.i.d. random variables. How does the ZRP behave when we condition the system to have a fixed density (*i.e.* average number of particles per site)? Under some conditions — described for example by Grosskinsky, Schütz and Spöhn (2003) and Janson (2012) — the zero-range process exhibits condensation.

We consider in this talk the non-homogeneous ZRP, introduced by Godrèche and Luck (2012), in which we first sample random fitnesses in every site of the system, before running a ZRP, where the occupation numbers $(Q_1, \ldots, Q_n)$ are independent, but not identical. A site with a larger fitness will likely contain more particles.

I will describe how the non-homogeneous ZRP behaves and under which conditions condensation occurs: this is an ongoing work, in collaboration with Peter Mörters (University of Bath) and Daniel Ueltschi (University of Warwick).

Uniform Sampling of Subshifts of Finite Type

Irène Marcovici, Institut Élie Cartan de Lorraine

Let us color the vertices of the lattice $\mathbb{Z}^d$ using a finite number of colors, with the constraint that some pairs of colors are not allowed for adjacent sites. What do "typical" configurations look like? To answer, we would like to be able to sample uniformly on the set of all possible configurations. The set of colorings of a lattice with forbidden patterns can be seen as a subshift of finite type (SFT), and "typical" configurations are then described by the measures of maximal entropy of the SFT. I will present some contributions to the sampling of these measures (joint work with Jean Mairesse). For a SFT on $\mathbb{Z}$, we provide constructions of the measure of maximal entropy starting from a sequence of i.i.d. random variables. For a multi-dimensional SFT, we introduce a sampling procedure based on probabilistic cellular automata. In a work in progress, I also study the extension of these notions to colorings of infinite regular trees. Work supported by the European INTEGER project.

Enumeration of Cubic Multigraphs on Orientable Surfaces

Markus Moßhammer, Technische Universität Graz

In recent years there have been various results in counting graphs embeddable on orientable surfaces, especially for planar graphs. For example the number of embeddable graphs with n vertices and m edges with $m < n$ is known for planar graphs but not for graphs embeddable on surfaces of higher genus. One way to count such graphs is a constructive decomposition resulting in the problem of counting cubic multigraphs on orientable surfaces.

In this talk we will show the asymptotic number of cubic multigraphs embeddable on an orientable surface of genus g to be asymptotically

$$c_g \gamma^n n^{5/2(g-1)-1} n!,$$

where c_g is a constant depending on the genus and the growth constant γ is independent of the genus. To do this, methods from analytic combinatorics and structural graph theory will be used as well as results from Gao on triangulations.

The Weighted Branching Process and the `Quicksort` Process

Uwe Rösler, Christian-Albrechts-Universität zu Kiel

The talk gives an overview of the weighted branching process. One may view the process as a Markov chain or a dynamical system indexed by a tree. Central objects will be stochastic fixed point equations and the relation to the running time analysis of stochastic divide-and-conquer algorithms. Finally a new result on the almost sure convergence of the discrete `Quicksort` process in Skorokhod metric is presented.

Profile of PATRICIA Tries

Wojciech Szpankowski, Purdue University
Joint work with Abram Magner

A PATRICIA trie is a trie in which non-branching paths are compressed. The external profile $B_{n,k}$, defined to be the number of leaves at level k of a PATRICIA trie on n nodes, is an important summarizing parameter, terms of which several other parameters of interest can be formulated. In this talk, we present our results giving precise asymptotics for the expected value and variance of $B_{n,k}$, as well as a central limit theorem with error bound, for PATRICIA tries on n i.i.d. binary strings generated by a memoryless source with bias $p > 1/2$ for k in the range where the profile grows polynomially with n. In this range, $\mathrm{E}[B_{n,k}] = \Theta(\mathrm{Var}[B_{n,k}])$, and both are of the form $\Theta(n^{\beta(\alpha)}/\sqrt{\log n})$, where $\alpha = k/\log n$ and the Θ hides bounded, periodic functions in $\log n$ whose Fourier series are explicitly known. The compression property leads to extra terms in the Poisson functional equations for the profile which are not seen in tries or digital search trees, resulting in Mellin transforms which are only implicitly given in terms of the moments of $B_{m,j}$ for various m and j. Thus, our derivations require information about the profile outside the range of interest. As an application of our results on the profile, we present a conjecture, with progress toward its proof, about the precise asymptotics of the typical height in asymmetric PATRICIA tries (the work on the height is joint with Michael Drmota). Our derivations rely on analytic techniques, including Mellin transforms, analytic depoissonization, and the saddle point method.

Berry-Esseen Bounds for Geometric and Combinatorial Random Graphs

Christoph Thäle, Ruhr-Universität Bochum
Joint work with Kai Krokowski, Anselm Reichenbachs and Matthias Schulte

We discuss two recent applications of the so-called Malliavin-Stein method for normal approximation in the framework of Poisson random measures and Rademacher sequences. Our first application deals with a class of edge-length functionals of the radial spanning tree of a Poisson point process. The latter arises when each point of a stationary Poisson point process in $\mathbb{R}^d$ is connected to its radial nearest neighbor. The second application is concerned with the triangle counting statistic in the classical Erdős-Rényi random graph model $G(n,p)$. We will demonstrate how the Malliavin-Stein technique can be used to find a Berry-Esseen bound if the success probability is of the form $p = \theta n^{-\alpha}$ with $\theta \in (0,1)$ and $\alpha \in [0,1)$.

Recurrence Function for Sturmian Words: Probabilistic Studies

Pablo Rotondo, Universidad de la República: **Part (I): A First Probabilistic Model**

Brigitte Vallée, Université de Caen: **Part (II): A Second Probabilistic Model**

The recurrence function measures the "complexity" of an infinite word u. It describes the possible occurrences of finite factors inside u together with the gaps between these occurrences. More precisely, the recurrence of factors of length n inside u is described by the integer $R_u(n)$ that is the smallest integer m for which every factor of u of length m contains all the factors of u of length n. The recurrence function of the infinite word u is then defined by the whole sequence $n \mapsto R_u(n)$.

This recurrence function is widely studied, notably in the case of Sturmian words, which are in a sense the simplest infinite words which are not eventually periodic. With each Sturmian word is associated an irrational number α, and many of the characteristics of the sturmian word $S(\alpha)$ depend on the continued fraction expansion of α; this is in particular the case for the recurrence function which is expressed in terms of the sequence of continuants $k \mapsto q_k(\alpha)$ of α.

Most of the classical studies on the recurrence function for Sturmian words deal with a *fixed* α, and the usual focus is put on *extreme* behaviours of the recurrence function. Here, we adopt "dual" approaches which are probabilistic : in the following two models, the slope α is uniformly drawn in the unit interval and gives rise to a random Sturmian word $S(\alpha)$. However, the role of the index n is different according to the model.

- Part (I). In the first model, we are interested in *particular* sequences of indices $k \mapsto n_k$ which have a *given* "position" with respect to the sequence of continuants $k \mapsto q_k(\alpha)$, and thus strongly depend on α; when α is *uniformly drawn* inside the unit interval, we perform a *probabilistic* study of this sequence for $k \to \infty$. We then better understand the role of the "position" in the recurrence function. This is a joint work with Valérie Berthé, Eda Cesaratto, Pablo Rotondo, Brigitte Vallée, Alfredo Viola.
- Part (II). In the second model, we draw the slope α uniformly in $[0,1]$ and the index n uniformly

in $[1..N]$, and independently on α. We perform a *probabilistic* study of the recurrence function of the word $S(\alpha)$ at index n, and we then let $N \to \infty$. This second model is completely natural when we wish to study the recurrence function for random pairs (α, n) where the slope α and the index n are independent. This is a joint work in progress with Pablo Rotondo and Brigitte Vallée.

The Number of Spanning Trees of Random 2-trees

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Joint work with Elmar Teufl

A 2-tree is obtained recursively by starting with a root triangle and, in each step of the construction, attaching a new triangle at an edge. The number of spanning trees of a random 2-tree was recently considered in the statistical physics literature by Xiao and Zhao, who made several conjectures on the growth of this quantity. Using the tools of analytic combinatorics, we study the asymptotic behaviour of the number of spanning trees of random 2-trees for different notions of randomness suggested by models of random trees.