ADVANCED TOPICS IN MATHEMATICAL LOGIC. SOMMER SEMESTER 2018.

LYUBOMYR ZDOMSKYY

The course will be devoted to combinatorial covering properties of topological spaces (sometimes called selection principles), with special emphasis on sets of reals, and their interplay with forcing. It is going to be self-contained, modulo basic knowledge of general topology and forcing.

At the beginning of each lecture I will distribute a short script of the previous one. If you have missed some lectures and would like to have the script please send me an e-mail.

Schedule.
Tuesday, Thursday 14:00-15:00. First lecture: 6.03.2018

Language: English.

What we have already done:

- *Lecture 1, 6.03.2018.* We have introduced all the covering properties the course is devoted to and discussed some basic implications. An overview of the course was given.

- *Lecture 2, 8.03.2018.* We have proved a characterization of the covering properties in terms of their continuous images into the Baire space and drawn some corollaries thereof.

- *Lecture 3, 13.03.2018.* We have discussed the covering properties of concentrated sets as well as other special subsets of reals. We have also proved that in the Cohen model, any set of ground model reals has the Rothberger property.

- *Lecture 4, 15.03.2018.* We have discussed the covering properties of scales and and drawn some corollaries thereof, e.g., that the Menger property is not preserved by finite products in a strong sense under $\mathfrak{b} = \mathfrak{d}$.

- *Lecture 5, 20.03.2018.* We have presented several non-preservation by products results under CH as well as constructed a non-meager Menger filter outright in ZFC.

- *Lecture 6, 22.03.2018.* This lecture was devoted to games associated to covering properties. In particular, we have proved that the existence of a winning strategy for the second player puts very strong restrictions on a space.

- *Lecture 7, 10.04.2018.* We have proved that a space is Menger iff the first player has no winning strategy in the Menger game on it, and started to prove the analogous result for Rothberger property.

- *Lecture 8, 12.04.2018.* We have finished proving that a space is Rothberger iff the first player has no winning strategy in the Rothberger game on it, and started to prove some aplications of these game characterizations of the properties of Menger and Rothberger.

- *Lecture 9, 17.04.2018.* We have proved a characterization of spaces which remain Hurewicz after adding Cohen reals. Also, we have derived several corollaries from the following fact which is to be proved next time: A Mathias forcing associated to a filter does not add dominating reals iff the filter has Menger covering property.

- *Lecture 10, 19.04.2018.* We have proved that a Mathias forcing associated to a filter does not add dominating reals iff the filter has Menger covering property.

- *Lecture 11, 24.04.2018.* A. Medini (the guest lecturer) presented some auxiliary results needed to characterize those $X \subset 2^\omega$ such that $\mathcal{K}(X)$, the space of all compact subsets of X with the Vietoris topology, is hereditarily Baire.

- *Lecture 12, 26.04.2018.* A. Medini (the guest lecturer) finished the proof of the fact that for $X \subset 2^\omega$, $\mathcal{K}(X)$ is hereditarily Baire iff $2^\omega \setminus X$ is Menger.

- *Lecture 13, 03.05.2018.* We have proved that a Mathias forcing associated to a filter is almost ω^ω-bounding in the sense of Shelah iff the filter has Hurewicz covering property.

- *Lecture 14, 08.05.2018.* We have characterized filters on ω whose Mathias forcing preserves ground model reals non-meager.

- *Lecture 15, 15.05.2018.* We have established some auxiliary results about games on filters needed to prove that the Mathias forcing for an analytic filter keeps ground model reals non-meager iff it is equivalent to the Cohen forcing .

- *Lecture 16, 17.05.2018.* We started to prove Con($\kappa = \mathfrak{b} < \mathfrak{a} = \kappa^+ = \mathfrak{c}$) following Brendle's strategy.

References

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[2] Hurewicz, W., *Über die Verallgemeinerung des Borellschen Theorems*, Math. Z. **24** (1925), 401–421.

[3] Hurewicz, W., *Über Folgen stetiger Funktionen,* Fund. Math. **9** (1927), 193–204.

[4] Just, W.; Miller, A.W.; Scheepers, M.; Szeptycki, P.J., *The combinatorics of open covers. II,* Topology Appl. **73** (1996), 241–266.

[5] Menger, K., *Einige Überdeckungssätze der Punktmengenlehre, Sitzungsberichte*. Abt. 2a, Mathematik, Astronomie, Physik, Meteorologie und Mechanik (Wiener Akademie) **133** (1924), 421–444.

[6] Scheepers, M., *Combinatorics of open covers. I. Ramsey theory,* Topology Appl. **69** (1996), 31–62.

[7] Todorčević, S., *Aronszajn orderings. Djuro Kurepa memorial volume,* Publ. Inst. Math. (Beograd) (N.S.) **57**(71) (1995), 29–46.

E-mail address: `lzdomsky@gmail.com`
URL: `http://dmg.tuwien.ac.at/zdomskyy/`