

# Curriculum Vitae

## Personal data

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born 2. August 1973 in Tulln, Lower Austria

Austrian citizenship, married, two daughters

## Training and Employments

- |                 |   |
|-----------------|---|
| 1989            | AFS exchange student in Australia (Koonung High School, Melbourne, Victoria)                                  |
| 06/1991         | Matura  |
| 10/1991–08/2002 | Study at Vienna University<br>at first Computer Science and Logic, since 10/1992 Mathematics                  |
| 06/1996–05/1997 | civil service (agriculture)   |
| 03/1998–05/2000 | programmer at BOSCH Telecom   |
| 05/2000–02/2001 | parental leave  |
| 06/2000         | Masters Degree in Mathematics   |
| 02/2001–08/2002 | Pre-Doc-position within a project of the Austrian Science Fund of Christian Krattenthaler                     |
| 08/2002         | Phd in Mathematics  |
| 09/2002–12/2003 | Post-Doc-Position at the LaBRI, Université Bordeaux 1,<br>within the ACE - European research training network |
| 01/2004–05/2004 | Post-Doc-Position within a project of the Austrian Science Fund of Michael Drmota                             |
| 06/2004–05/2006 | Assistent at the Institute for Statistics and Decision Support, Universität Wien                              |

since 06/2006

Post-Doc-Position within an NFN project of the Austrian  
Science Fund of Christian Krattenthaler

## Teaching

Within my employment at the Institute for Statistics and Decision Support I taught:

- Exercises Mathematics I and II
- Exercises Probability I and II
- Exercises Statistics II
- Exercises nonparametric statistics

## Computer Skills

The programming language of my choice is ANSI Common Lisp. Within my scientific research I make heavy use of several computer algebra systems, like Mathematica, Maxima, Maple, MuPAD und Axiom. Meanwhile I became one of the leading developers of Axiom.

## Further Skills and Interests

Languages

English and French fluent, Spanish

Pedagogics

I am strongly interested in alternative approaches to education in the sense of Montessori, Piaget and Wild. From 1997 until 2002 I was responsible for youth groups in different parishes.

March 30, 2007

# Research Report

Primarily I am interested in enumerative Combinatorics. In the following I would like to give a brief overview of my past research. All of my articles save [3, 18, 14] are available electronically at <http://www.mat.univie.ac.at/~rubey>.

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## 1 Research Articles

### 1.1 Spanning Trees

In my Diploma-thesis [1] I considered the problem of counting the number of spanning trees of a given graph. The first part is devoted to related objects that are in bijection with the spanning trees of a graph. Among those we consider spanning forests, perfect matchings, recurrent configurations of the sandpile model and Eulerian trails.

The second part is concerned with combinatorial methods of enumeration. For example, I study a Prüfer-like coding of generalised lexicographic products of graphs.

In the last part I present the matrix-tree-theorem, and then go on to describe various methods for determining the eigenvalues of the Laplacian matrix of a graph. First I describe what can be inferred about the eigenvalues of graphs that are generated from other graphs as products or other operations. A short section is devoted to the relationship of the spectrum of a graph and his group of automorphisms. Finally I give a method to guess

the eigenvalues of graphs that can be defined as the restriction of an infinite regular grid.

A bit related to my Diploma-thesis is my article [8] together with Srećko Brlek, Michel Mendès France and Michael Robson on ‘Cantorian matrices’. Essentially it is about the number of edge-colourings of the complete graph, that contain a certain coloured subgraph. This subgraph can be found schematically in Figure 1, where two edges should have the same colour if they are incident to the same vertex in the bottom row.

## 1.2 Dynamical Systems

One section of my Diploma-thesis is concerned with the sandpile model on graphs. The dynamical structure of this model can be described by a Markov chain, whose recurrent configurations are in bijection with the spanning trees of the given graph. I retake this topic in the manuscript [18], and derive an asymptotic formula for the number of maximal avalanches in a certain family of graphs. An example for such a graph is given in Figure 2.

In the article ‘Pattern-specific neural network design’, together with H. Schweng, Karl E. Kürten and Karl W. Kratky [3] we discovered heuristics to increase the capacity of Hopfield networks.

## 1.3 Optimisation

In joint work with Immanuel Bomze and Florian Frommlet [12] we study the problem of minimising a quadratic form over the  $\ell^1$ -ball. One of my contributions was the observation that a certain matrix could be interpreted as the Laplacian of a graph and is thus positive semidefinite.

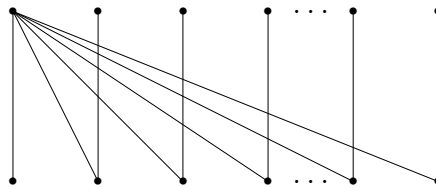


Figure 1.

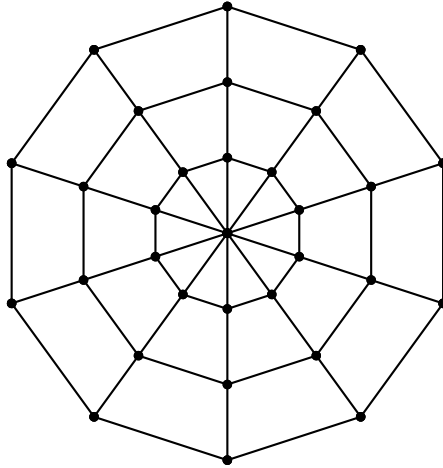


Figure 2.

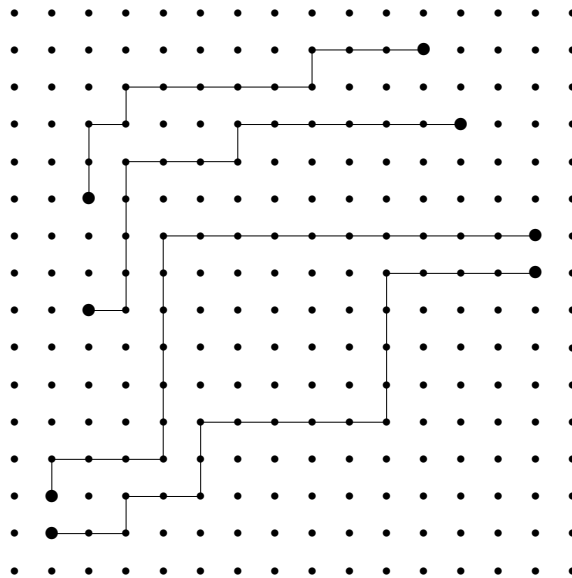


Figure 3.

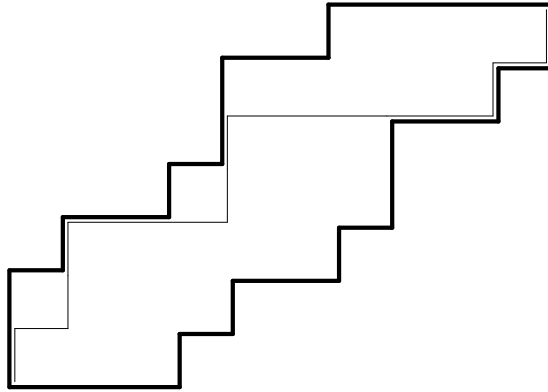


Figure 4.

## 1.4 Lattice Paths

### Lattice Paths in a Ladder

In my Doctoral thesis I considered families of non-intersecting lattice paths in  $\mathbb{Z}^2$  with steps in directions north and east and given starting and ending points. An example for such a family with four paths is given in Figure 3. Together with Christian Krattenthaler [6] we were able to prove a formula for the number of such families, that have a given number of north-east turns which lie all in a given region, a so-called (one-sided) ladder. Here a ladder is a subset of  $\mathbb{Z}^2$  that is bounded by two arbitrary non-intersecting lattice paths.

In a further article I was able to show that the corresponding generating function for a single path,

$$\sum_{w \text{ lattice path}} x^{\#\text{north-east turns in } w}$$

is log-concave. This confirmed a special case of a conjecture of Aldo Conca.

My article about equidistributed statistics of lattice paths in a ladder heads in a different direction. We consider the number of ‘maxima’ and ‘minima’ of a single lattice path with north and east steps in a ladder. A ‘minimum’ (resp. ‘maximum’) is an east step that runs along the south (resp. north) boundary of the ladder. For example, the lattice path in Figure 4 has 3 minima and 2 maxima.

Guo-Niu Han discovered that the number of lattice paths with  $k$  ‘minima’ and  $l$  ‘maxima’ equals the number of lattice paths with  $l$  ‘minima’ and  $k$  ‘maxima’. In [14] I was able to give a simple bijective proof of a refinement of this fact.

Furthermore I considered the following related statistics: on the one hand the number of vertical steps of the lattice paths that run along the east boundary, on the other hand the number of vertical steps that run along the west boundary of the ladder. For example, the lattice path in Figure 4 has 3 left contacts and 2 right contacts.

It turns out that the number of lattice paths with  $k$  left contacts and  $l$  minima equals the number of lattice paths with  $l$  right contacts and  $k$  maxima.

Summarising we have

$$\sum_{P \text{ lattice path}} x^{\#\text{maxima}} y^{\#\text{minima}} = \sum_{P \text{ lattice path}} x^{\#\text{minima}} y^{\#\text{maxima}}$$

and

$$\sum_{P \text{ lattice path}} x^{\#\text{left contacts}} y^{\#\text{minima}} = \sum_{P \text{ lattice path}} x^{\#\text{right contacts}} y^{\#\text{maxima}}.$$

Surprisingly, in the latter case the interpretation of the lattice paths as bases of a matroid plays a crucial role. In fact, we only need the independence of the Tutte-polynomial of the ordering of the ground set of the matroid.

### Asymptotic Analysis of the number of Lattice Paths

In the section about ‘vicious walkers’ of my dissertation I determine the asymptotics for  $m \rightarrow \infty$  of the number of families of non-intersecting lattice paths with north-east and south-east steps, where the starting points are located on the positive  $y$ -axis and the endpoints on the vertical line through  $(m, 0)$ . An example with  $m = 12$  can be found in Figure 5. Thus I was able to generalise a result of Christian Krattenthaler.

In ‘Transcendence of generating functions of walks on the slit plane’[7] we consider random walks from  $(0, 0)$  to  $(1, 0)$  on the slit plane, i.e., in the grid  $\mathbb{Z}^2$  with the negative  $x$ -axis removed. An example for such a walk with steps in

$$\mathfrak{S} = \{(-1, -2), (-1, 1), (-1, 2), (1, -2), (1, 1), (1, 2)\}$$

can be found in Figure 6. Employing asymptotic methods I was able to show for a large class of step sets  $\mathfrak{S}$  that the corresponding generating function

$$\sum_{w \text{ random walk with steps in } \mathfrak{S}} x^{\#\text{steps in } w}$$

is transcendent if and only if  $\mathfrak{S}$  contains no step with height larger than one. Equivalently, the random walk can not cross the negative  $x$ -axis without stepping on it. This result confirms a conjecture of Mireille Bousquet-Mélou.



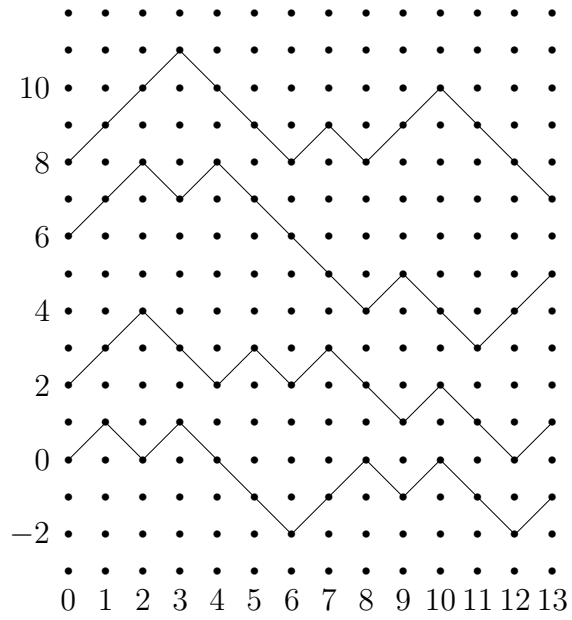


Figure 5.

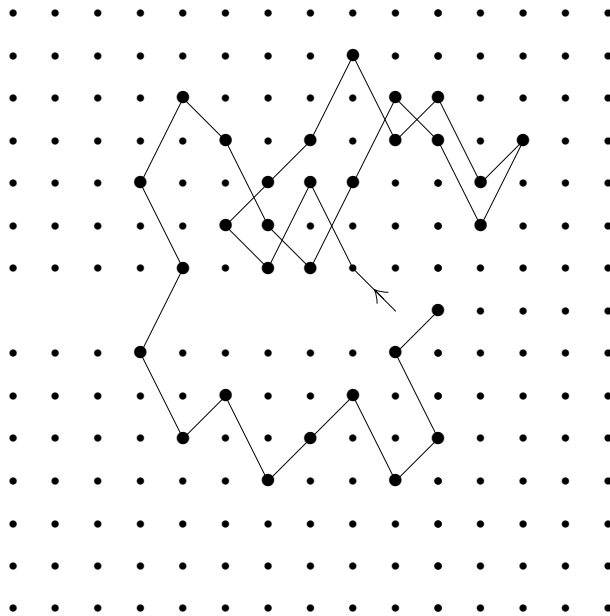


Figure 6.

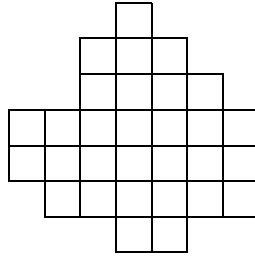


Figure 7.

## Faulhabers Coefficients

In the seventeenth century Johann Faulhaber studied formulae for the sums of the first  $n$   $k^{\text{th}}$  powers. He discovered that these formulae are always polynomials in  $n(n+1)$ . The coefficients of these polynomials bear his name.

Recently, several authors discovered various  $q$ -analogues of Faulhaber's formulae, i.e., expressions with an additional variable  $q$ , that converge towards Faulhaber's when  $q$  tends to 1.

In [19] I generalise the combinatorial interpretation of Faulhaber's coefficients by Ira Gessel and Xavier Viennot as number of certain families of non-intersecting lattice paths to the new setting. Thus I answered a question posed by Victor Guo and Jiang Zeng. Together with Victor Guo and Jiang Zeng [10] we could complete the picture.

## 1.5 Tableaux and Polyominoes

In the article "A 'nice' bijection for a content formula for skew semistandard Young tableaux" [2] I use a modified Jeu de Taquin procedure to answer a question posed by Richard Stanley.

In [15] I use yet another variant of jeu de taquin to prove an interesting symmetry property of fillings of so-called moon-polyominoes, as introduced by Jakob Jonnson. A moon-polyomino is a convex polyomino with the additional property that for any two columns one must be contained in the other. An example for such a polyomino can be found in Figure 7.

For a given moon polyomino and given  $k$  and  $n$  we now count in how many ways we can put  $n$  balls into the boxes, such that the longest north-east chain (satisfying certain natural properties) is shorter than  $k$ . It turns out that this number depends only on the sequence of column-heights of the polyomino, and not on its particular shape.

The methods I developed in this article I could finally apply also to related questions in the hyperoctahedral group.[17]

## 1.6 Symbolic Computing

Frequently the computer is an indispensable tool to discover conjectures or disprove them, that is, to do experimental mathematics. In this spirit I have written some programs that may well be interesting for combinatorialists as well as for physicists.

### Guessing Formulae for sequences of numbers

One of the principal problems of combinatorics is to determine the size of a family of objects. In the simplest case, this number depends only on a single parameter  $n$ . Often it is possible to enumerate the objects for small  $n$  and then try to guess the general formula.

A few years ago the first tools that help guessing such formulae appeared. One of the best known is probably Neil Sloanes ‘on-line encyclopedia of integer sequences’, a program that checks whether the given sequence occurs in a database of currently roughly 130000 known sequences.

My program **Guess** heads in a different direction. On the one hand it employs known algorithms such as interpolation and Padé approximation. Furthermore it implements less well known algorithms that enable us also to guess formulae satisfying an algebraic differential equation. On the other hand it also contains a new method that is able to recognise quickly formulae of the form

$$n \mapsto (a + bn)^n r(n) \quad \text{or} \quad n \mapsto \binom{a + bn}{n} r(n),$$

where  $r$  is a rational function. Apart from this the program can also cope with  $q$ -analogs. For example, it can guess differential equations for generating functions  $f(x)$  of the form

$$p(f(x), f(qx), f(q^2x), \dots) = 0,$$

where  $p$  is a polynomial with coefficients in  $\mathbb{D}[q]$ , and  $\mathbb{D}$  is a – in principle arbitrary – computable integral domain. Moreover, my program is significantly faster than other programs, such as **GFUN** by Bruno Salvy and Paul Zimmermann or **Rate** by Christian Krattenthaler.

This program is described in [16] and can be tried out online at <http://wiki.axiom-developer.org/GuessingFormulas>.

### Combinatorial Species

In joint work with Ralf Hemmecke I designed a program to deal with André Joyal’s combinatorial species. In abstract terms, a combinatorial species  $F$

is a functor from the category of finite sets and bijections into the category of finite sets and bijections. In more practical terms, a species  $F$  is a set of labeled objects, which is closed under relabeling. As an example consider the species of partitions, which produces for any finite set  $U$  all set partitions of  $U$ .

Since the produced set of objects is closed under relabeling, it makes sense to consider also the set of unlabeled objects, that is, the set of isomorphismtypes. To obtain this set, one makes the individual elements in  $U$  indistinguishable. It is easy to see that one obtains as isomorphismtypes of the species of partitions exactly the integer partitions.

To any species we associate several generating functions. It is immediately clear that one would like to have access to the (exponential) generating function for the number of labeled objects, and the (ordinary) generating function for unlabeled objects.

Now there are many natural transformations that build new species from old. For example, we might be interested in the sum, the product or the composition of two given species. As an example, we can describe the species of partitions as *sets of nonempty sets*. The important point is that these combinatorial operations correspond to the operations on formal power series, albeit not always in a trivial way.

Our program thus enables us to generate and count easily and efficiently combinatorial objects. Of course, there are other facilities that can do that, for example `MuPAD-Combinat`, `Combstruct` or Frank Ruskey's `Combinatorial Object Server`. However, concerning flexibility and mathematical exactness, our program is even after only half a year of development time in some ways more advanced than these programs. It is freely distributed via `svn://svn.risc.uni-linz.ac.at/hemmecke/combinat`.

## 2 Referee Reports

for the journals Journal of Combinatorial Theory, Series A, Analysis of Algorithms, Discrete Mathematics and Discrete Mathematics & Theoretical Computer Science. Furthermore I wrote some reviews for Mathematical Reviews.

## 3 Organisational Experience

Together with Ralf Hemmecke from RISC/Universität Linz I organised a workshop on Computer Algebra und Diskrete Mathematik, which took place in April 2006. We were able to attract renowned speakers from Austria and abroad: Nicolas Thiéry (Université Paris Sud), Petr Hliněný (Masaryk University, Brno), Bernhard Gittenberger (Technische Universität Wien) und Carsten Schneider (RISC/ Universität Linz).

One of the results of this workshops is the package which implements André Joyals theory of combinatorial species.

Also for this year we organised a workshop, which will be concerned with the implementation of symmetric functions. As speakers we invited Harald Friepertinger (Universität Graz) and, again Nicolas Thiéry gewinnen.

## Presentations

Below a list of some selected presentations I have given.

- The  $h$ -vector of a ladder determinantal ring cogenerated by  $2 \times 2$  minors is log-concave, *49th Séminaire Lotharingien de Combinatoire*, Ellwangen, October 2002
- Marcheurs méchants, *Groupe de travail de Combinatoire Énumérative et de Génération Aléatoire*, Bordeaux, November 2002
- A nice bijection for a content formula for skew semistandard Young tableaux, *Groupe de travail de Combinatoire Énumérative et de Génération Aléatoire*, Bordeaux, February 2003
- Méthodes combinatoires d'énumération des arbres couvrants dans certaines familles de graphes, *Groupe de travail de Combinatoire Énumérative et de Génération Aléatoire*, Bordeaux, March 2003
- The generating function for Walks on the Slit Plane is transcendental, if you can cross the slit without touching it, *Summer School on Enumerative Combinatorics*, Linköping, Juli 2003
- Autour d'une jolie bijection, *Groupe de travail de Combinatoire Énumérative et de Génération Aléatoire*, Bordeaux, October 2003
- 'Nice' bijections for paths in a ladder, *52th Séminaire Lotharingien de Combinatoire*, Ottrott, March 2004
- Transcendence of generating functions of walks on the slit plane *Mathematics and Computer Science III*, Wien, September 2004
- Symmetry properties of statistics of lattice paths in ladders, *Noon lecture at the Department of Applied Mathematics of the Faculty of Mathematics and Physics*, Karls Universität, Prag, Mai 2006
- Increasing and decreasing sequences in fillings of moon Polyominoes *Sixth Czech-Slovak International Symposium on Combinatorics, Graph Theory, Algorithms and Applications*, Wien, Juli 2006
- Increasing and decreasing sequences in fillings of moon Polyominoes *Seminár z teórie grafov*, Comenius Universität, Bratislava, Oktober 2006

## 4 Publications and Preprints

### Thesis

- [1] Martin Rubey. Counting spanning trees. Master’s thesis, Universität Wien, 2000. <http://www.univie.ac.at/~rubey/diplom.ps.gz>.
- [2] Martin Rubey. *Nonintersecting lattice paths in Combinatorics, Commutative Algebra and Statistical Mechanics*. PhD thesis, Universität Wien, 2002. <http://www.univie.ac.at/~rubey/Dissertation.ps.gz>.

### Articles in Journals and Proceedings

- [3] Martin Anderle, H. Schweng, Karl E. Kürten, and Karl W. Kratky. Pattern-specific neural network design. *Journal of Statistical Physics*, 81(3–4):843–849, 1995.
- [4] Martin Rubey. Comment on ‘Counting nonintersecting lattice paths with turns’ by C. Krattenthaler. *Séminaire Lotharingien Combinatoire*, (34), 2001. Comment on paper B34i.
- [5] Martin Rubey. A “nice” bijection for a content formula for skew semistandard Young tableaux. *Electronic Journal of Combinatorics*, 9(1):Research Paper 18, 13 pp. (electronic), 2002. arXiv:math.CO/0011099.
- [6] Christian Krattenthaler and Martin Rubey. A determinantal formula for the Hilbert series of one-sided ladder determinantal rings. In *Algebra, arithmetic and geometry with applications (West Lafayette, IN, 2000)*, pages 525–551. Springer, Berlin, 2004. arXiv:math.AC/0106076.
- [7] Martin Rubey. Transcendence of generating functions of walks on the slit plane. In *Mathematics and computer science. III*, Trends Math., pages 49–58. Birkhäuser, Basel, 2004. arXiv:math.CO/0405188.
- [8] Srečko Brlek, Michel Mendès France, Michael Robson, and Martin Rubey. Cantorian Tableaux and Permanents. *l’Enseignement Mathématique*, 50(3-4):287–304, 2004. arXiv:math.CO/0308081.
- [9] Martin Rubey. The  $h$ -vector of a ladder determinantal ring cogenerated by  $2 \times 2$  minors is log-concave. *Journal of Algebra*, 292(2):303–323, 2005. arXiv:math.AC/0205212.

- [10] Victor J. W. Guo, Martin Rubey, and Jiang Zeng. Combinatorial Interpretations of the  $q$ -Faulhaber and  $q$ -Salie Coefficients. *Journal of Combinatorial Theory, Series A*, 113(7):1501–1515, 2006. arXiv:math.CO/0506274.
- [11] Martin Rubey. Extended Rate, more GFUN. In *Proceedings of the Fourth Colloquium on Mathematics and Computer Science Algorithms, Trees, Combinatorics and Probabilities*, Discrete Mathematics and Theoretical Computer Science, DMTCS, 2006. extended abstract of [16].
- [12] Immanuel M. Bomze, Florian Frommlet, and Martin Rubey. Improving SDP bounds for minimizing quadratic functions over the 11-ball. *Optimization Letters*, 1(1):49–59, 2007. arXiv:math.OA/0503174.
- [13] Martin Rubey. Increasing and Decreasing Sequences in Fillings of Moon Polyominoes. In *Proceedings of the 19th International Conference on Formal Power Series and Algebraic Combinatorics*, Nankai University, Tianjin, China, 2007. extended abstract of [15].

## Preprints

- [14] Martin Rubey. Equidistributed statistics on paths in a ladder. *Preprint*, 2005.
- [15] Martin Rubey. Increasing and Decreasing Sequences in Fillings of Moon Polyominoes. Submitted to *Advances in Applied Mathematics*. arXiv:math.CO/0604140.
- [16] Martin Rubey. Extended Rate, more GFUN. Submitted to *Journal of Symbolic Computation*, 2007. arXiv:math.CO/0702086.
- [17] Martin Rubey. Triangulations, crossings and nestings in the hyperoctahedral group. *Preprint*.

## Manuscripts not intended for Publication

- [18] Martin Rubey. A note on maximal avalanches on the generalized wheel. 2003.
- [19] Martin Rubey. A combinatorial interpretation of Guo and Zeng’s  $q$ -Faulhaber coefficients. *Preprint*, 2005. arXiv:math.CO/0503114.