# CANONICAL FUNCTIONS: A PROOF VIA TOPOLOGICAL DYNAMICS

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Abstract. Canonical functions are a powerful concept with numerous applications in the study of groups, monoids, and clones on countable structures with Ramsey-type properties. In this short note, we present a proof of the existence of canonical functions in certain sets using topological dynamics, providing a shorter alternative to the original combinatorial argument. We moreover present equivalent algebraic characterisations of canonicity.

## 1. Introduction

When $f\colon (\mathbb{Q};<) \to (\mathbb{Q};<)$ is any function from the order of the rational numbers to itself, then there are arbitrarily large finite subsets of $\mathbb{Q}$ on which $f$ "behaves regularly"; that is, it is either strictly increasing, strictly decreasing, or constant. A direct (although arguably unnecessarily elaborate) way to see this is by applying Ramsey's theorem: two-element subsets of $\mathbb{Q}$ are colored with three colors according to the local behavior of $f$ on them (this yields, by the infinite version of Ramsey's theorem, even an infinite set on which $f$ behaves regularly, but this is beside the point for us). In particular, it follows that the closure of the set $\{\beta\, f\, \alpha \mid \alpha, \beta \in \mathrm{Aut}(\mathbb{Q};<)\}$ in $\mathbb{Q}^{\mathbb{Q}}$, equipped with the pointwise convergence topology, contains a function which behaves regularly everywhere. This function of regular behavior is called canonical.

More generally, a function $f\colon \Delta \to \Lambda$ between two structures $\Delta, \Lambda$ is called canonical when it behaves regularly in an analogous way, that is, when it sends tuples in $\Delta$ of the same type (in the sense of model theory, as in [Hod97]) to tuples the same type in $\Lambda$ [BPT13, BP14, BP11]. Similarly as in the example above, canonical functions can be obtained from $f$, in the fashion stated above, when $\Delta$ has sufficient Ramsey-theoretic properties (for example, the Ramsey property) and when $\Lambda$ is sufficiently small (for example $\omega$-categorical) [BPT13, BP14, BP11].

The concept of canonical functions has turned out useful in numerous applications: for classifying first-order reducts they are used in [Aga16, Pon13, PPP$^+$14, BPP15, BJP16b, AK16, LP15], for complexity classification for constraint satisfaction problems (CSPs) in [BMPP16,BW12,BP15a,BJP16a,KP16], for decidability of meta-problems in the context

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of the CSPs in [BPT13], for lifting algorithmic results from finite-domain CSPs to CSPs over infinite domains in [BM16], for lifting algorithmic results from finite-domain CSPs to homomorphism problems from definable infinite structures to finite structures [KKOT15], and for decidability questions in computations with atoms in [KLOT16]. Most of these applications are covered by a survey article published shortly after their invention [BP11].

As indicated above, the technique is available for a function $f\colon \Delta \to \Lambda$ whenever $\Delta$ is a Ramsey structure and $\Lambda$ is $\omega$-categorical, and the existence of canonical functions in the set $\overline{\{\beta\, f\, \alpha \mid \alpha \in \mathrm{Aut}(\Delta), \beta \in \mathrm{Aut}(\Lambda)\}} \subseteq \Lambda^{\Delta}$ was originally shown under these conditions by a combinatorial argument [BPT13, BP14, BP11]. By the Kechris-Pestov-Todorcevic correspondence [KPT05], a structure $\Delta$ is Ramsey (with respect to colorings of embeddings) if and only if its automorphism group $\mathrm{Aut}(\Delta)$ is *extremely amenable*, meaning that every continuous action of it on a compact Hausdorff space has a fixed point. Moreover, by the theorem of Ryll-Nardzewski, Engeler, and Svenonius, two tuples in a countable $\omega$-categorical structure have the same type if and only if they lie in the same orbit with respect to the componentwise action of its automorphism group on tuples, and a countable structure is $\omega$-categorical if and only if its automorphism group is oligomorphic. Therefore both the definition of canonicity as well as the above-mentioned conditions implying their existence in sets of the form $\overline{\{\beta\, f\, \alpha \mid \alpha \in \mathrm{Aut}(\Delta), \beta \in \mathrm{Aut}(\Lambda)\}}$ can be formulated in the language of permutation groups.

It is therefore natural to ask for a perhaps more elegant proof of the existence of canonical functions via topological dynamics, reminiscent of the numerous proofs of combinatorial statements obtained in this fashion (cf. the survey [Ber06] for Ergodic Ramsey theory; [Kec14] mentions some applications of extreme amenability). In this short note, we present such a proof. The proof was discovered by the authors at the Workshop on Algebra and CSPs at the Fields Institute in Toronto in 2011, where it was also presented (by the second author), but has so far not appeared in print. We use the occasion of this note to present various equivalent characterisations of canonicity of functions that facilitate their use and better explain their significance.

## 2. Canonicity

We use the notation $\mathbf{G} \curvearrowright X$ to denote a permutation group $\mathbf{G}$ acting on a set $X$. We make the convention that if $f\colon X \to Y$ is a function and $t = (t_1, \ldots, t_k) \in X^k$, where $k \geq 1$, then $f(t) := (f(t_1), \ldots, f(t_k)) \in Y^k$ denotes the $k$-tuple obtained by applying $f$ componentwise.

The following is an algebraic formulation of Definition 6 in [BPT13].

**Definition 1.** Let $\mathbf{G} \curvearrowright X$ and $\mathbf{H} \curvearrowright Y$ be permutation groups. A function $f\colon X \to Y$ is called *canonical with respect to* $\mathbf{G}$ *and* $\mathbf{H}$ if for every $k \geq 1$, $t \in X^k$, and $\alpha \in \mathbf{G}$ there exists $\beta \in \mathbf{H}$ such that $f\, \alpha(t) = \beta\, f(t)$.

Hence, functions that are canonical with respect to $\mathbf{G}$ and $\mathbf{H}$ induce for each integer $k \geq 1$ a function from the orbits of the componentwise action of $\mathbf{G}$ of $X^k$ to the orbits of the componentwise action of $\mathbf{H}$ on $Y^k$.

In order to formulate properties equivalent to canonicity we require some topological notions. We consider the set $Y^X$ of all functions from $X$ to $Y$ as a topological space equipped with the topology of pointwise convergence, i.e., the product topology where $Y$ is taken to be discrete. When $S \subseteq Y^X$, then we write $\overline{S}$ for the closure of $S$ in this space. In particular, when $\mathbf{G} \curvearrowright X$ is a permutation group, then $\overline{\mathbf{G}}$ is the closure of $\mathbf{G}$ in $X^X$. Note that $\overline{\mathbf{G}}$ might no longer be a group, but it is still a monoid with respect to composition of functions. For example, in the case of the full symmetric group $\mathbf{G} = \mathrm{Sym}(X)$ consisting of all permutations of $X$, $\overline{\mathbf{G}}$ is the transformation monoid of all injections in $X^X$.

A permutation group $\mathbf{G} \curvearrowright X$ is called *oligomorphic* if for each $k \geq 1$ the componentwise action of $\mathbf{G}$ on $X^k$ has finitely many orbits. For oligomorphic permutation groups we have the following equivalent characterisations of canonicity.

**Proposition 1.** *Let $\mathbf{G} \curvearrowright X$ and $\mathbf{H} \curvearrowright Y$ be permutation groups, where $\mathbf{H} \curvearrowright Y$ is oligomorphic. Then for any function $f \colon X \to Y$ the following are equivalent.*

(1) *$f$ is canonical with respect to $\mathbf{G}$ and $\mathbf{H}$;*
(2) *for all $\alpha \in \mathbf{G}$ we have $f\alpha \in \overline{\mathbf{H}f} := \overline{\{\beta f \mid \beta \in \mathbf{H}\}}$;*
(3) *for all $\alpha \in \mathbf{G}$ there are $e_1, e_2 \in \overline{\mathbf{H}}$ such that $e_1 f \alpha = e_2 f$.*

A stronger condition is to require that for all $\alpha \in \mathbf{G}$ there is an $e \in \overline{\mathbf{H}}$ such that $f\alpha = ef$. To illustrate that this is strictly stronger, already when $\mathbf{G} = \mathbf{H}$, we give an explicit example.

**Example 2** (thanks to Trung Van Pham)**.** Let $\mathbf{G} := \mathrm{Aut}(\mathbb{Q}; <)$. Note that $(\mathbb{Q}; <)$ and $(\mathbb{Q} \setminus \{0\}; <)$ are isomorphic, and let $f$ be such an isomorphism. Then $f$, viewed as a function from $\mathbb{Q} \to \mathbb{Q}$, is clearly canonical with respect to $\mathbf{G}$ and $\mathbf{G}$. But $f$ does not satisfy the stronger condition above: there is no $e \in \overline{\mathbf{G}}$ such that $f\alpha = ef$. To see this, choose $b, c \in \mathbb{Q}$ such that $f(b) < 0 < f(c)$. By transitivity there exists an $\alpha \in \mathbf{G}$ such that $\alpha(b) = c$. Note that $0 < f\alpha(b) < f\alpha(c)$. Morever, the image of $f\alpha$ equals the image of $f$, and hence any $e \in \overline{\mathbf{G}}$ such that $f\alpha = ef$ must fix 0. Since $e$ must also preserve $<$, it cannot map $f(b) < 0$ to $f\alpha(b) > 0$. Hence, there is no $e \in \overline{\mathbf{G}}$ such that $f\alpha = ef$. $\square$

In Proposition 1, the implications from (1) to (2) and from (3) to (1) follow straightforwardly from the definitions. For the implication from (2) to (3) we need a lift lemma, which is in essence from [BPP14]. This lemma has been applied frequently lately [BJP16a, BP16, BM16], in various slightly different forms. We need a yet different formulation here; since the lemma is a consequence of the compactness of a certain space which we need in any case for the canonisation theorem in Section 3, we present its proof.

Let $\mathbf{H} \curvearrowright Y$ be a permutation group, and let $f, g \in Y^X$, for some $X$. We say that $f = g$ *holds locally modulo* $\mathbf{H}$ if for all finite $F \subseteq X$ there exist $\beta_1, \beta_2 \in \mathbf{H}$ such that $\beta_1 f \restriction_F = \beta_2 g \restriction_F$. We say that $f = g$ *holds globally modulo* $\mathbf{H}$ *(modulo* $\overline{\mathbf{H}}$*)* if there exist $e_1, e_2 \in \mathbf{H}$ ($e_1, e_2 \in \overline{\mathbf{H}}$, respectively) such that $e_1 f = e_2 g$.

Of course, if $f = g$ holds globally modulo $\overline{\mathbf{H}}$, then it holds locally modulo $\mathbf{H}$. On the other hand, if $f = g$ holds locally modulo $\mathbf{H}$, then it need not hold globally modulo $\mathbf{H}$. To see this, let $f(x, y) \colon \omega^2 \to \omega$ be an injection, set $g := f(y, x)$, and let $\mathbf{H}$ be the group of all permutations of $\omega$. Then $f = g$ holds locally modulo $\mathbf{H}$, but not globally. However, there exist injections $e_1, e_2 \in \omega^\omega$ such that $e_1\, f = e_2\, g$, so $f = g$ holds globally modulo $\overline{\mathbf{H}}$. This is true in general, as we see in the following lift lemma.

**Lemma 3.** *Let $\mathbf{H} \curvearrowright Y$ be an oligomorphic permutation group, let $I$ be an index set, and let $X_i$ be a set for every $i \in I$. Let $f_i, g_i$ be functions in $Y^{X_i}$ such that $f_i = g_i$ holds locally modulo $\mathbf{H}$ for all $i \in I$. Then there exist $e, e_i \in \overline{\mathbf{H}}$ such that $e\, f_i = e_i\, g_i$ holds globally for all $i \in I$.*

To prove Lemma 3, it is convenient to work with a certain compact Hausdorff space that we also use for the canonisation theorem in Section 3. Let $\mathbf{H} \curvearrowright Y$ be a permutation group, and $X$ be a set. On $Y^X$, define an equivalence relation $\sim$ by setting $f \sim g$ if $f \in \overline{\mathbf{H}\, g}$, i.e., if $f = g$ holds locally modulo $\mathbf{H}$; here, transitivity and symmetry follow from the fact that $\mathbf{H}$ is a group. The following has essentially been shown in [BP15b] (though for the finer equivalence relation of global equality modulo $\mathbf{H}$), but we give an argument for the convenience of the reader since it is used so often (cf. for example [BJ11, BOP15, BPP14, BKO$^+$16]).

**Lemma 4.** *If $\mathbf{H} \curvearrowright Y$ is oligomorphic, then the space $Y^X/_\sim$ is a compact Hausdorff space.*

*Proof.* We represent the space in such a way that this becomes obvious. Extend the definition of the equivalence relation $\sim$ to all spaces $Y^F$, where $F \subseteq X$. When $F$ is finite, then $Y^F/_\sim$ is finite and discrete, because $\mathbf{H}$ is oligomorphic. Hence, the space

$$\prod_{F \in [X]^{<\omega}} Y^F/_\sim$$

is compact. The mapping from $Y^X/_\sim$ into this space defined by

$$[g]_\sim \mapsto \big([g \restriction_F]_\sim \mid F \in [X]^{<\omega}\big)$$

is well-defined. In fact, it is a homeomorphism onto a closed subspace thereof, since the topology on $Y^X/_\sim$ is precisely given by the behavior of functions on finite sets, modulo the equivalence $\sim$. Hence, $Y^X/_\sim$ is indeed a compact Hausdorff space. $\square$

*Proof of Lemma 3.* For simplicity of notation, assume that the $X_i$ are countable; then $Y^{X_i}$ is a metric space (otherwise, we would have to work with more general topological notions than sequences). We have $f_i \in \overline{\mathbf{H}\, g_i}$; so let $(\beta_i^j\, g_i)_{j\in\omega}$ be a sequence converging to $f_i$ for all $i \in I$. Setting $X := Y$ we see that $X^X/_\sim$ is compact by Lemma 4. Therefore, the set

$$\{([\delta]_\sim, ([\delta\, \beta_i^j]_\sim)_{i\in I}) \mid j \in \omega, \delta \in \mathbf{H}\}$$

is a subset of a compact space, $(X^X/_\sim) \times (X^X/_\sim)^I$. Hence, it has an accumulation point $([e]_\sim, ([e_i]_\sim)_{i\in I})$. Clearly, $e, e_i \in \overline{\mathbf{H}}$ for all $i \in I$, and the functions $e_i$ prove the lemma. $\square$

The implication from (2) to (3) in Proposition 1 now is a direct consequence of Lemma 3.

## 3. Canonisation

The following is the *canonisation theorem*, first proved combinatorially in [BPT13] in a slightly more specialized context.

**Theorem 5.** *Let* $\mathbf{G} \curvearrowright X$, $\mathbf{H} \curvearrowright Y$ *be permutation groups, where* $\mathbf{G}$ *is extremely amenable and* $\mathbf{H}$ *is oligomorphic, and let* $f\colon X \to Y$. *Then*

$$\overline{\mathbf{H}\, f\, \mathbf{G}} := \overline{\{\beta\, f\, \alpha \mid \alpha \in \mathbf{G}, \beta \in \mathbf{H}\}}$$

*contains a canonical function with respect to* $\mathbf{G}$ *and* $\mathbf{H}$.

*Proof.* The space $\overline{\mathbf{H}\, f\, \mathbf{G}}/_\sim$ is a closed subspace of the compact Hausdorff space $Y^X/_\sim$ from Lemma 4, and hence is a compact Hausdorff space as well. We define a continuous action of $\mathbf{G}$ on this space by

$$(\alpha, [g]_\sim) \mapsto [g\, \alpha^{-1}]_\sim\ .$$

Clearly, this assignment is a function, it is a group action, and it is continuous. Since $\mathbf{G}$ is extremely amenable, the action has a fixed point $[g]_\sim$. Any member $g$ of this fixed point is canonical: whenever $\alpha \in \mathbf{G}$, then $[g\, \alpha]_\sim = [g]_\sim$, which is the definition of canonicity. □

In applications of Theorem 5 (e.g., in [Aga16, Pon13, PPP+14, BPP15, BJP16b, AK16, LP15, BMPP16, BW12, BP15a, BJP16a, KP16, BPT13, BM16, KLOT16, BP11]), one usually needs the following special case of the above situation. It states, roughly, that whenever we have a finite arity function $f$ on a set, and an oligomorphic extremely amenable permutation group $\mathbf{G}$ on the same set, then we can obtain from $f$ and $\mathbf{G}$, using composition and topological closure, a canonical function whilst retaining finite information about $f$.

In the following statement, for $m \geq 1$ we write $\mathbf{G}^m$ for the natural action of $\mathbf{G}^m$ on $X^m$ given by $((\alpha_1, \ldots, \alpha_m), (x_1, \ldots, x_m)) \mapsto (\alpha_1(x_1), \ldots, \alpha_m(x_m))$. Moreover, we denote the pointwise stabilizer of $c^1, \ldots, c^n \in X^m$ in $\mathbf{G}^m$ by $(\mathbf{G}^m, c^1, \ldots, c^n)$.

**Corollary 6.** *Let* $\mathbf{G} \curvearrowright X$ *be an oligomorphic extremely amenable permutation group. Let* $f\colon X^m \to X$ *for some* $m \geq 1$, *and let* $c^1, \ldots, c^n \in X^m$ *for some* $n \geq 1$. *Then there exists*

$$g \in \overline{\mathbf{G}\, f\, \mathbf{G}^m}$$

*such that*

- $g$ *agrees with* $f$ *on* $\{c^1, \ldots, c^n\}$, *and*
- $g$ *is canonical with respect to the groups* $(\mathbf{G}^m, c^1, \ldots, c^n)$ *and* $\mathbf{G}$.

*Proof.* The group $\mathbf{G}^m$ is obviously extremely amenable. Moreover, it is known that so is any stabilizer of it (in fact, every open subgroup; cf. [BPT13]). The statement therefore follows from Theorem 5. □

## 4. Open Problems

Is there a converse of Theorem 5 in the sense that extreme amenability of $\mathbf{G}$ is equivalent to some form of the statement of the canonisation theorem? More precisely, we ask the following question.

**Question 7.** *Let* $\mathbf{G}$ *be the automorphism group of a countably infinite linearly ordered structure with domain* $X$*. Is it true that* $\mathbf{G}$ *is extremely amenable if and only if for all oligomorphic permutation groups* $\mathbf{H} \curvearrowright Y$ *and every* $f\colon X \to Y$ *we have that* $\overline{\mathbf{H}\, f\, \mathbf{G}}$ *contains a function that is canonical with respect to* $\mathbf{H}$ *and* $\mathbf{G}$*?*

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