Perfect matchings in cubic graphs

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based on a joint work with Edita Máčajová

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- **Part II:** Three perfect matchings with empty intersection

Perfect matchings in cubic graphs

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Every bridgeless cubic graphs contains a perfect matching.

- A cubic graphs has two disjoint perfect matchings $\Leftrightarrow$ 3-edge-colourable.
- Every two perfect matchings in a non-3-edge-colourable graph have an edge in common.

Perfect matchings in cubic graphs - conjectures

Conjecture (Fan & Raspaud, 1994) F&RC

Every bridgeless cubic graphs contains three perfect matchings with no edge in common.

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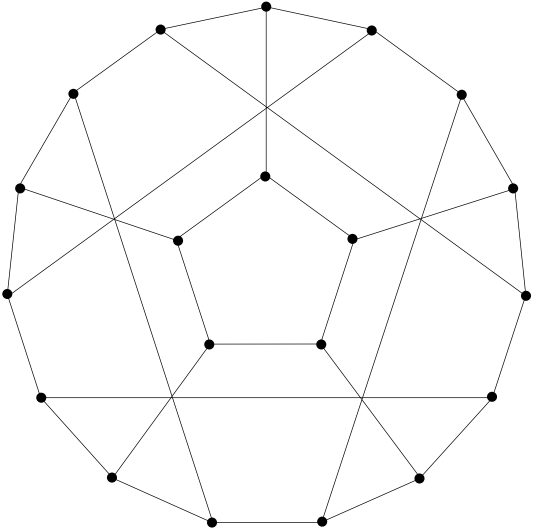
Every bridgeless cubic graphs contains a family of six perfect matchings which together cover each edge exactly twice.

Six perfect matchings in the flower snark I_5



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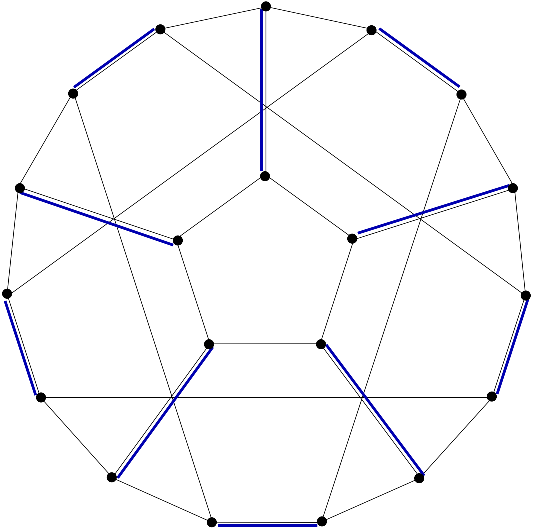

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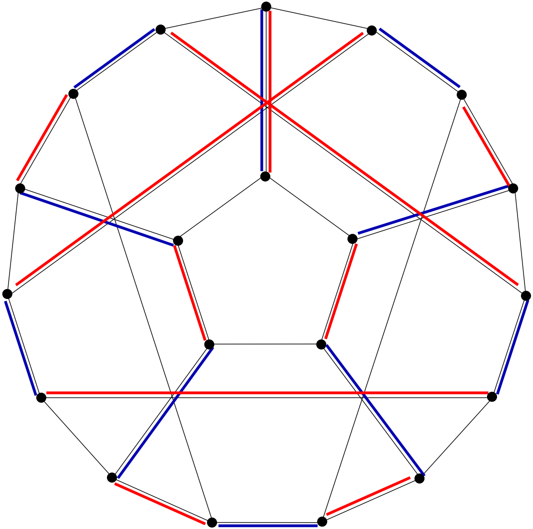

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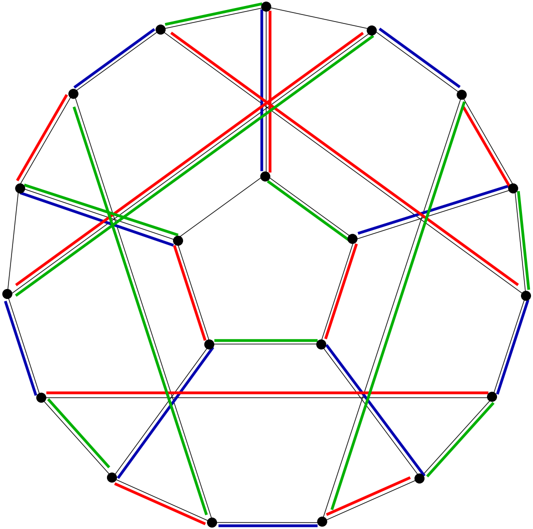

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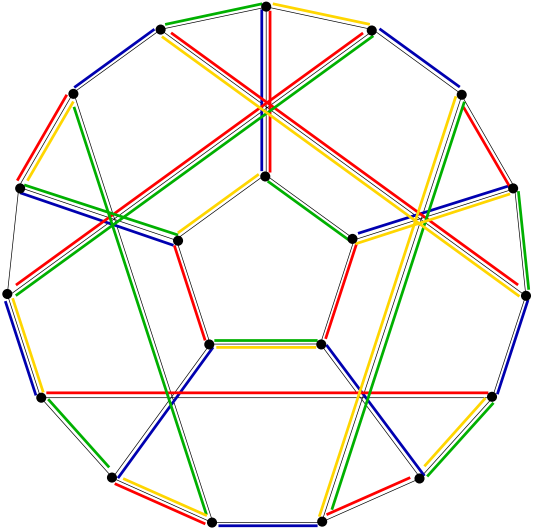

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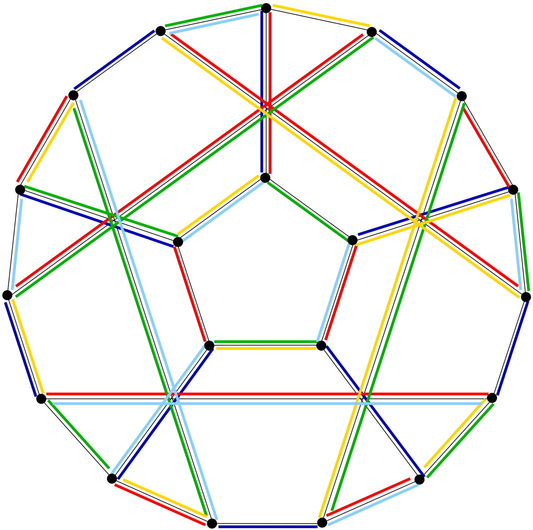

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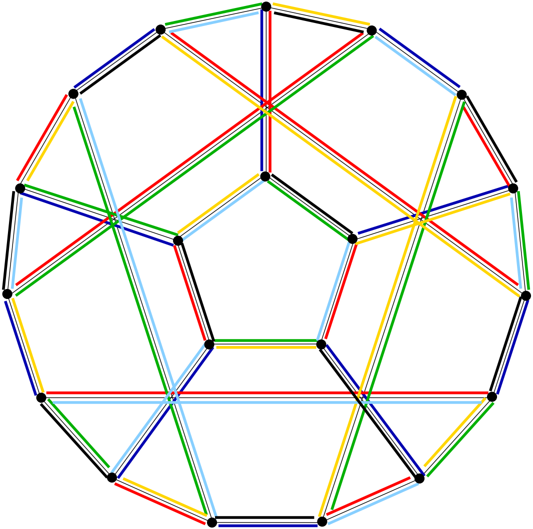

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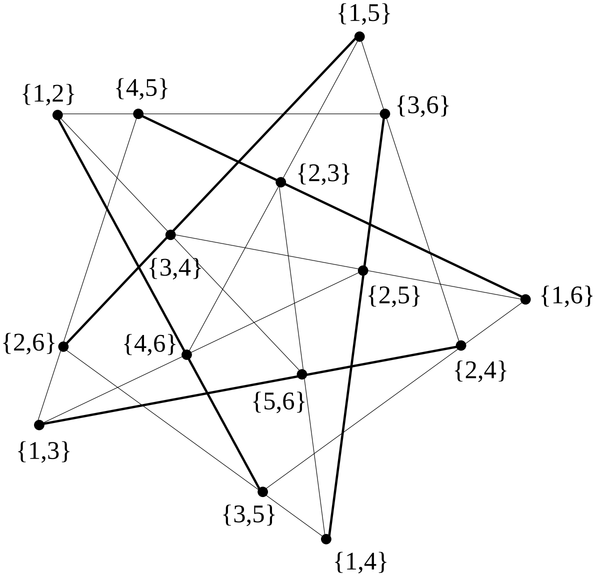
Theorem (Seymour, 1977)

If subtraction is allowed, then the constant function **2** *can be so obtained.*

FC and the Cremona-Richmond configuration 15_3



Related conjectures

Conjecture (Weak Version of Fulkerson's Conjecture)

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For every bridgeless cubic graph there exists a constant k *and* $3k$ *perfect matchings such that each edge is in edge is in* k *of them.*

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- $\exists k \forall G$ $\exists 3k$ PM s.t. every edge is in k PM ... **???** OPEN
- $\forall G \exists k$ $\exists 3k$ PM s.t. every edge is in k PM ... ✓ YES

Covering all edges by perfect matchings

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Berge's conjecture remains open even if 5 is replaced by any fixed k.

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- Berge's Conjecture $\Rightarrow$ $m_5(G) = 1$

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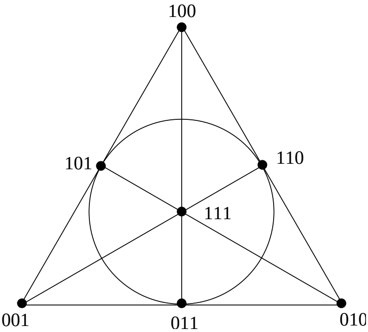
FC = 6-PMC ⇔ 5-PMC ⇒ 4-PMC⇒ 3-PMC = **F&RC**

Fano colourings of cubic graphs



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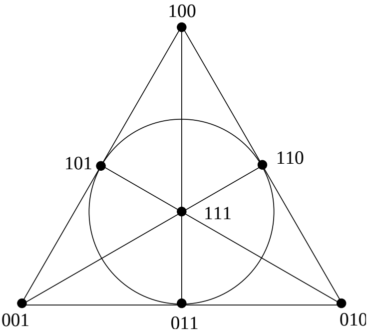

Fano colouring – proper edge-colouring of a cubic graph

- colours – points of the Fano plane
- around each vertex the colours form a line

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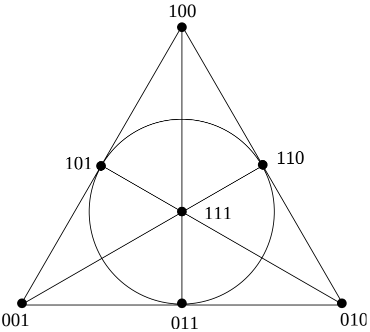

Fano colouring – proper edge-colouring of a cubic graph

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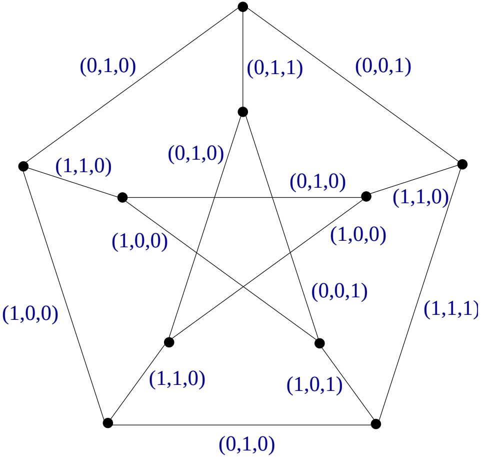
F_i-colouring – colouring using at most i lines of the Fano plane

F_5-colouring of the Petersen graph



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Theorem (Máčajová & S., 2005)

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4-Line Conjecture (Máčajová & S., 2005) F_4C

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Theorem (Máčajová & S., 2005)

F&RC=3-PMC is equivalent to the 4-Line Conjecture.

F_5-colourings

FC $=$ 6-PMC $\Leftrightarrow$ 5-PMC $\Rightarrow$ 4-PMC $\Rightarrow$ 3-PMC $=$ **F&RC**

F&RC $\Leftrightarrow$ F_4C $\Rightarrow$ F_5C $\Rightarrow$ F_6C $\equiv$ **TRUE**

F_5-colourings

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Theorem (Kaiser, Raspaud, 2007)

Every bridgeless cubic graph of oddness ≤ 2 admits an F_5-colouring.

F_4-colourings: Theorem

Theorem (Máčajová & S., 2008+)

Every bridgeless cubic graph of oddness ≤ 2 admits an F_4-colouring.

Equivalently:

Every bridgeless cubic graph of oddness ≤ 2 has three perfect matchings with no edge in common.

Proof of Theorem

I. G has oddness 0 ... ✓

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Call the triple $\mathcal{M} = (G, F, M)$ a mesh

Proof: meshes

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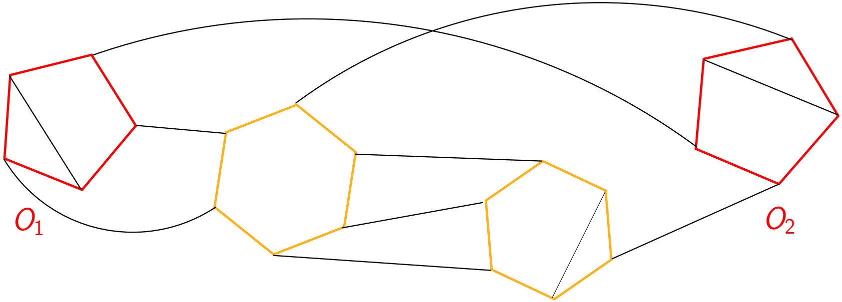

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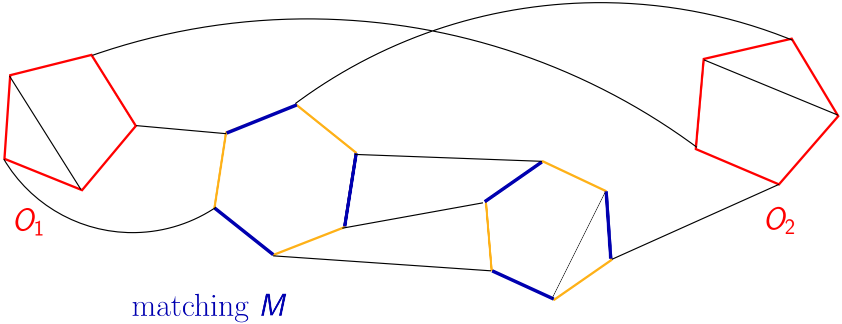
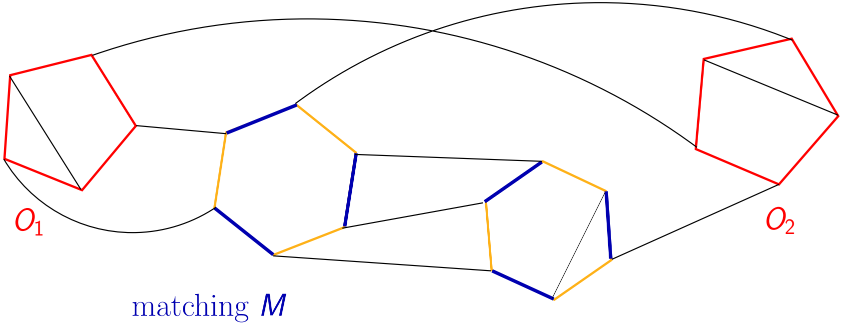


Proof: meshes

mesh ≅ partial 3-edge-colouring – non-coloured edges are red



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- different components are independent chains.

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Proposition

If a mesh $\mathcal{M} = (G, F, M)$ has two independent transversal chains, then G contains two perfect matchings F_1 and F_2 such that $F_1 \cap F_2 \cap F = \emptyset$.

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Aim: To prove that every mesh is good.

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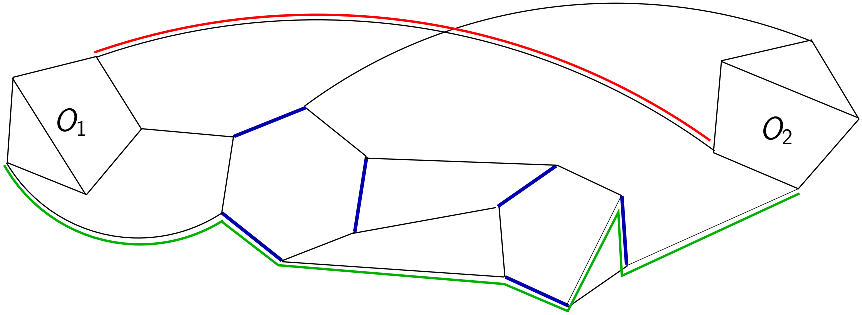

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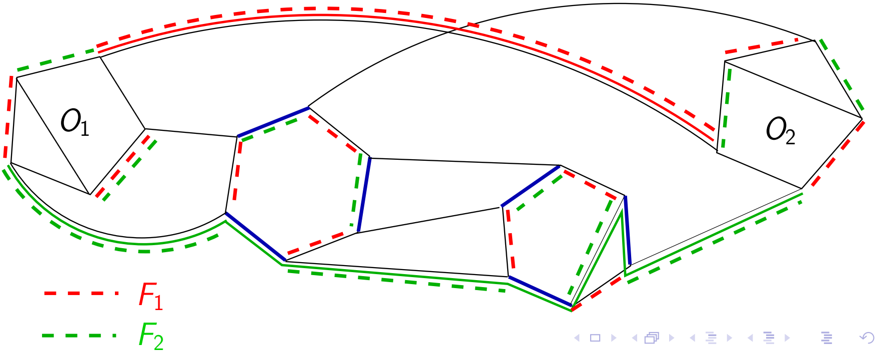
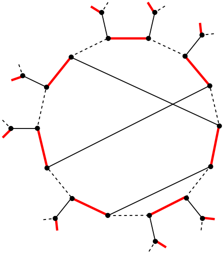
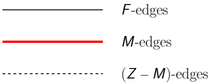


Necessary condition: I. Reduction

Necessary condition: dissolving a circuit of $C = G - F$





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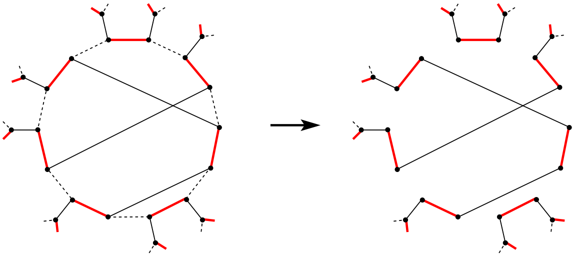

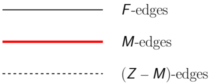

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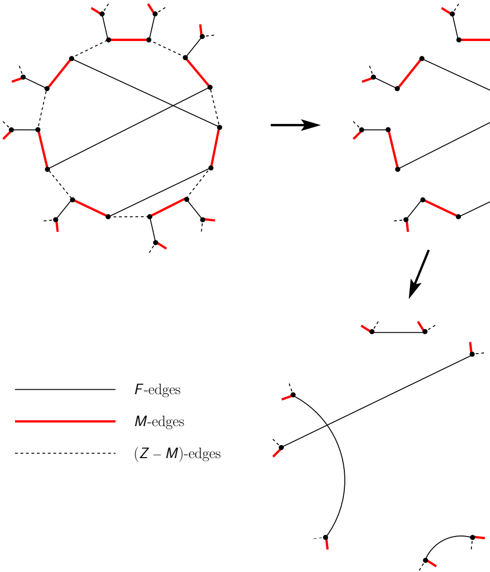

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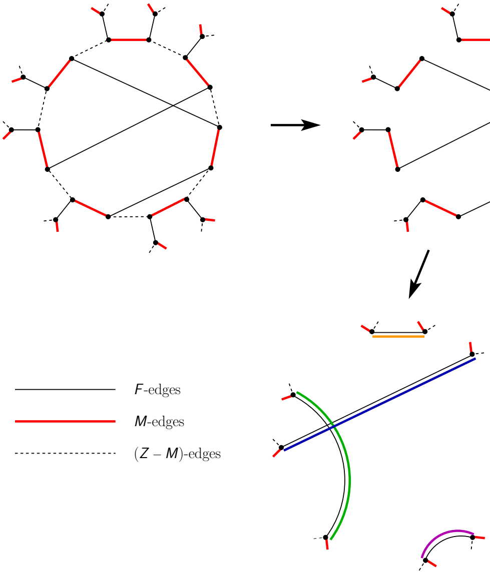

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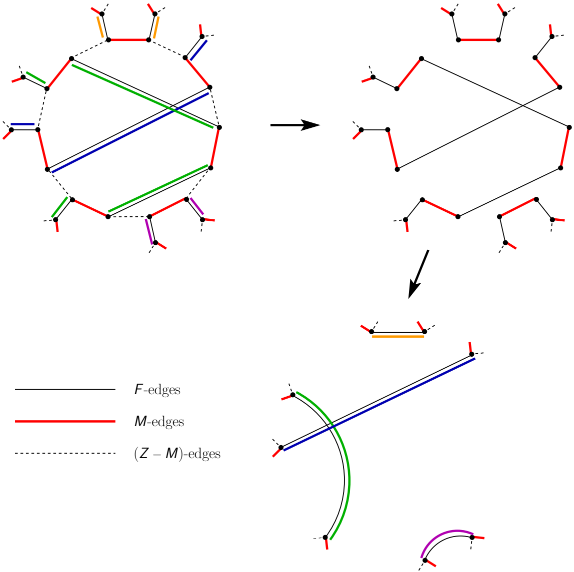

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We obtain a transformation

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Claim

If a reduction $\mathcal{M}'$ of $\mathcal{M}$ is good, then $\mathcal{M}$ is good.

Reduction: dissolving a circuit of $C = G - F$

We obtain a transformation

$G \rightarrow G'$ $F \rightarrow F'$ $M \rightarrow M'$

$\mathcal{M} = (G, F, M) \rightarrow \mathcal{M}' = (G', F', M')$

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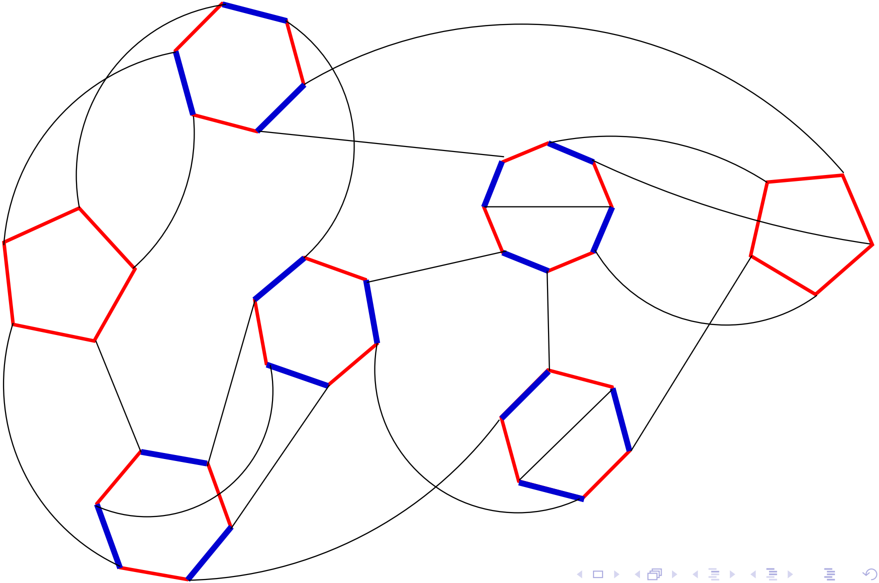
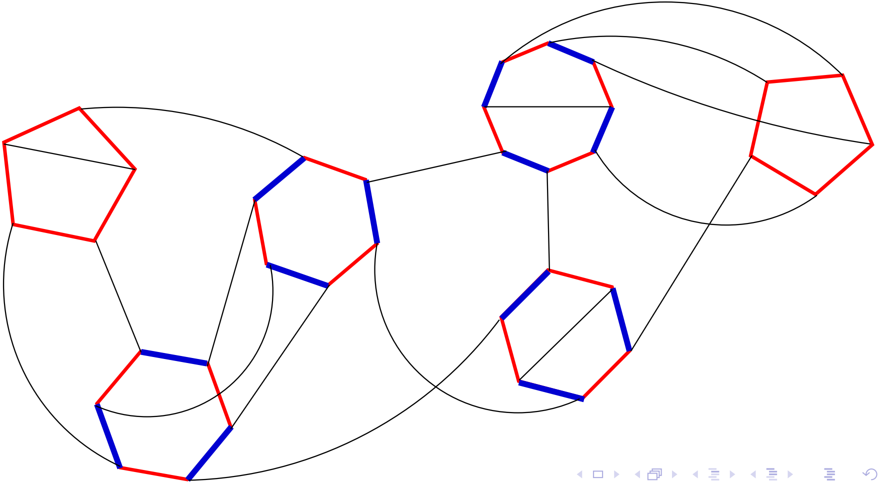
Corollary

It is enough to deal with primitive meshes.

Primitive meshes



Primitive meshes



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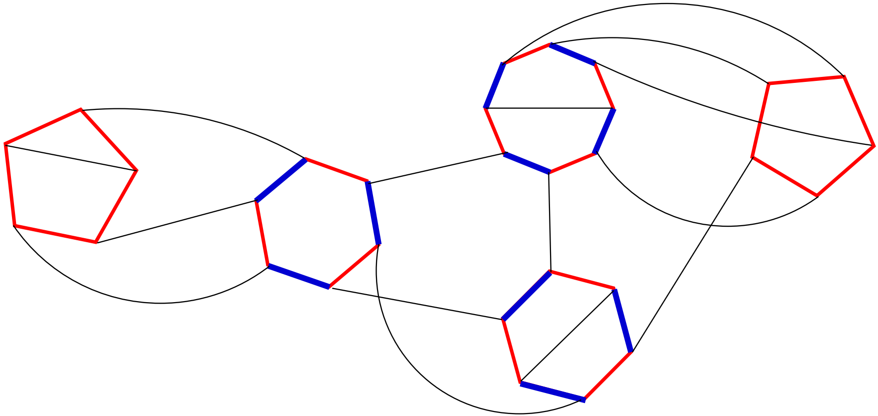

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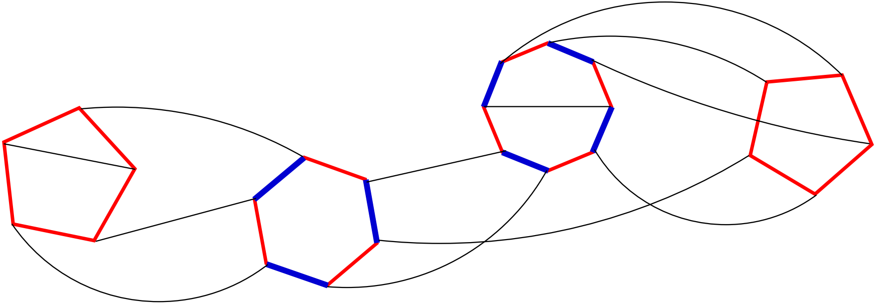

Primitive meshes: linear ordering of circuits

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Furthermore, this ordering is independent of the choice of M.

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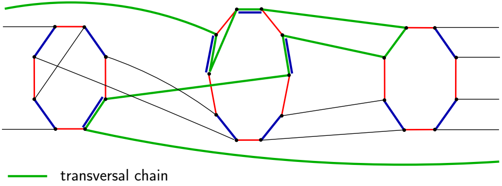

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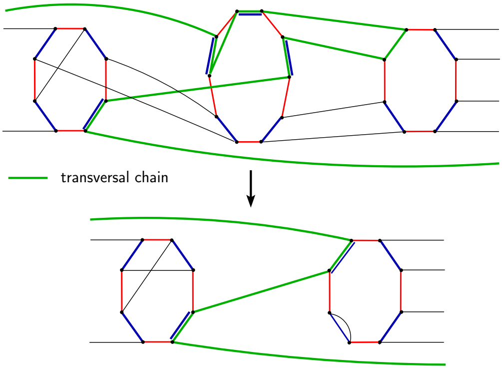

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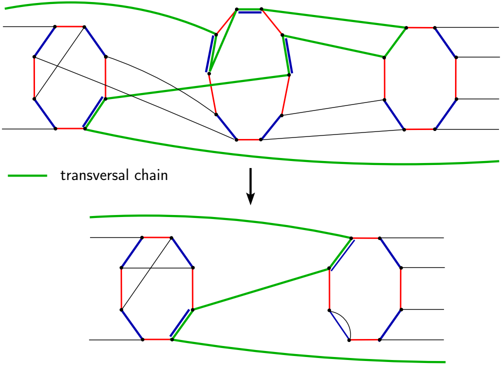

2. *Every local $(F \cup M)$-chain intersects only two consecutive circuits.*

Necessary condition: II. Construction of chains – transfer

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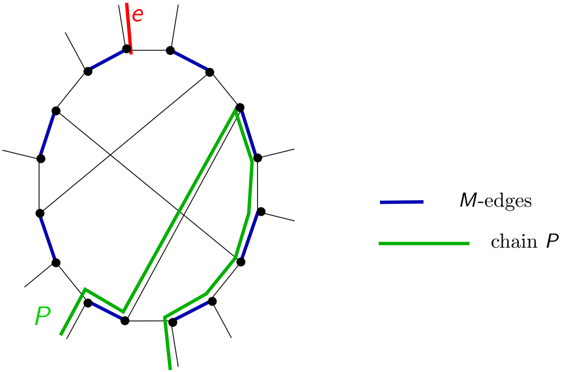

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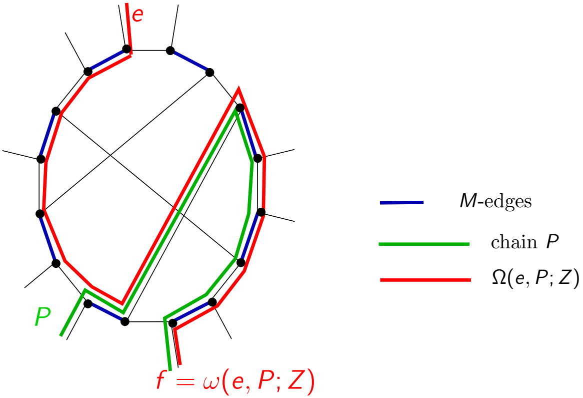
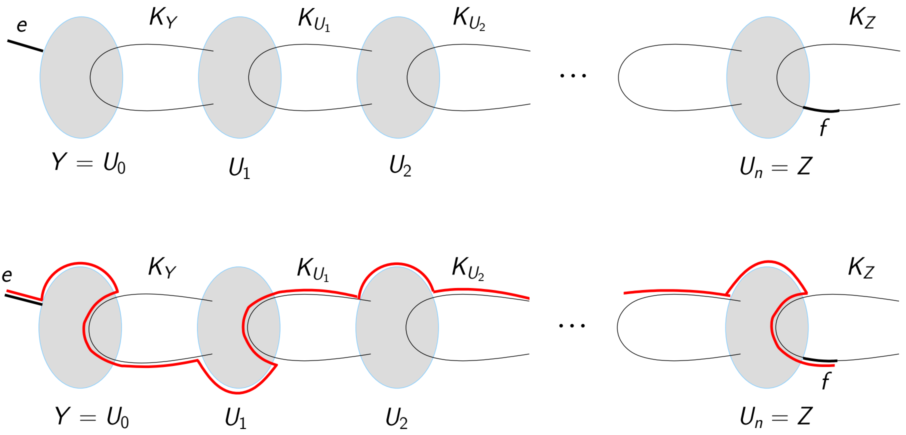
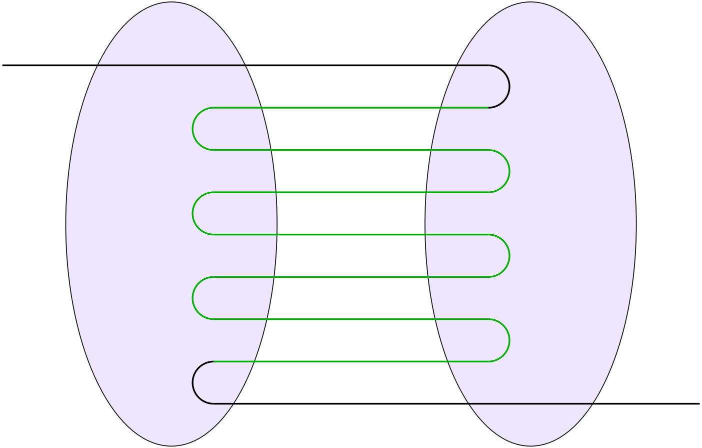


Multiple transfers: tubes

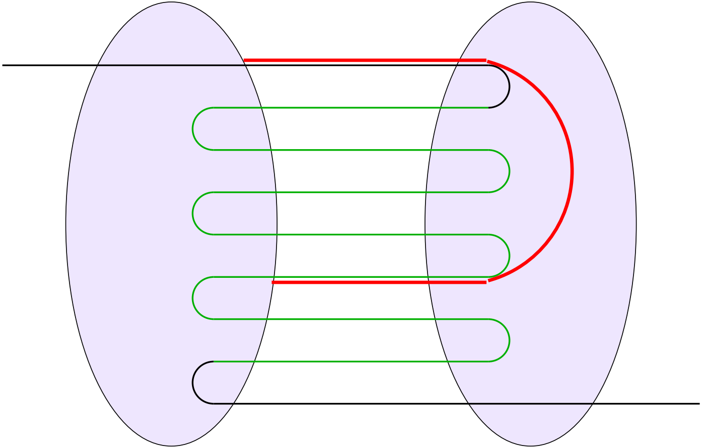


$\mathbf{K} = (K_U; Y \preceq U \preceq Z)$ $\quad f = \omega(e, \mathbf{K})$ $\quad$ ——— $\Omega(e, \mathbf{K})$

Smoothing



Smoothing: Case 1



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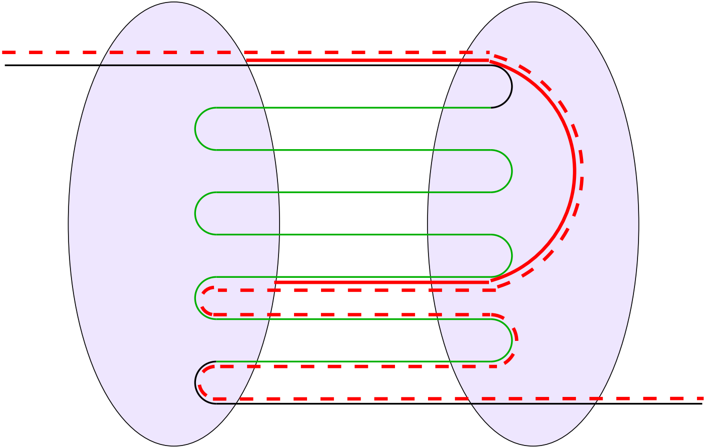
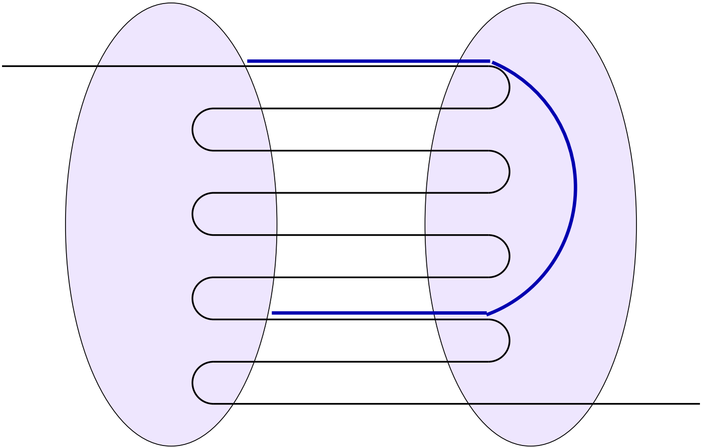
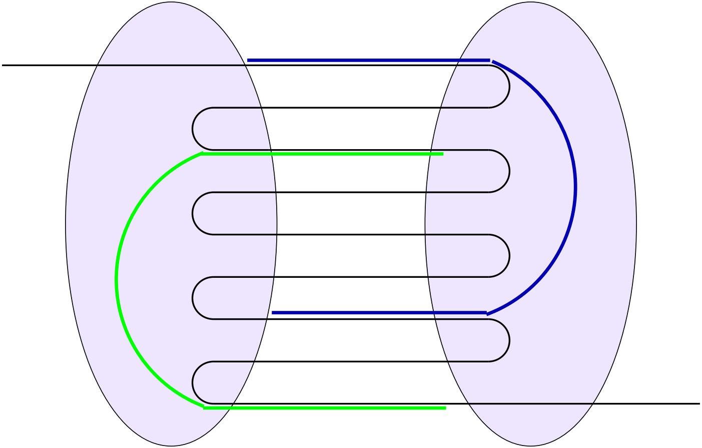


Smoothing: Case 2



Smoothing: Case 2



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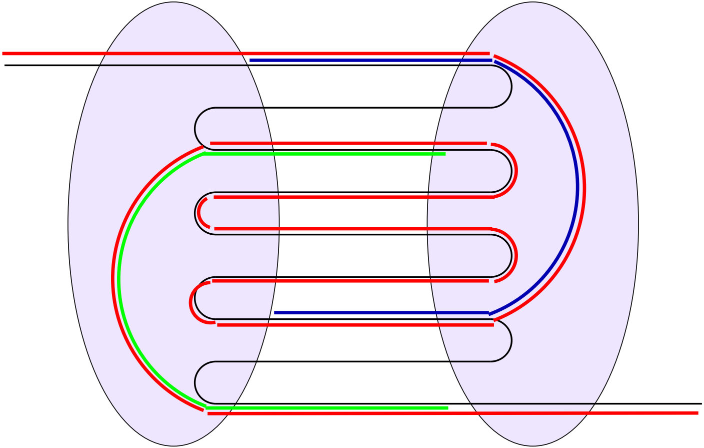
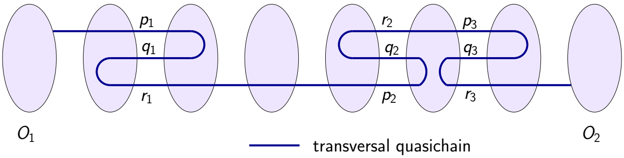


Final construction: transfer graph



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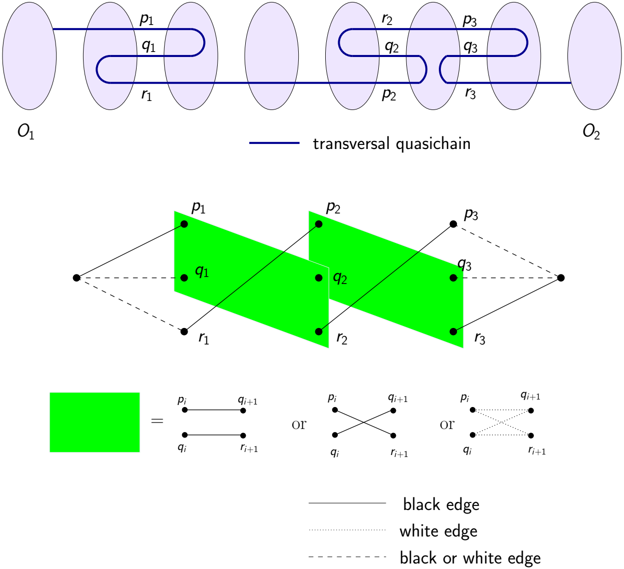

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Proposition

The transfer graph contains two internally disjoint increasing O_1-O_1-paths which together use at most one white edge from each class.

These two paths give rise to two independent transversal chains.

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Corollary

Every mesh on a bridgeless cubic graph is good.

□

Open problems

1. Higher oddness?

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Open problems

1. Higher oddness?
2. Other classes of cubic graphs?
3. Fulkerson Conjecture for oddness two?

Thank you!