Technische Universität Wien
Institut für Diskrete Mathematik und Geometrie
Forschungsgruppe Konvexe und Diskrete Geometrie

Einladung

zu einem

Vortrag

von Herrn

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mit dem Titel

"Convergence (in shape) of iterated Steiner symmetrizations"

Donnerstag, 14. Juni 2012, 10:30 Uhr

Institut für Diskrete Mathematik und Geometrie
Besprechungsraum, Turm A, 7. Stock
1040 Wien, Wiedner Hauptstraße 8-10

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Convergence (in shape) of iterated Steiner symmetrizations

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To fix Steiner's proof of the isoperimetric problem, W. Gross constructed, given a convex body K, a sequence of directions $\{u_n\}$ in order to minimize the perimeter and make the corresponding Steiner symmetrizations of K converge, in the Hausdorff distance, to the ball K^* centered at the origin and having the same volume as K.

Peter Mani was the first to understand (in 1986) that the convergence of the successive Steiner symmetrizations of a convex body K to K^* holds *almost surely*.

He conjectured that this happens also if we consider successive Steiner symmetrizations of a *compact* set. The conjecture was confirmed in 2006 by van Schaftingen. More recently we gave another proof of the conjecture. It is based on the following result.

Theorem *Let F be a measurable set having finite measure and P a symmetric probability on S^{N-1} which is strictly positive on each open set. If the directions are independent and random with respect to P, then almost surely the iterated Steiner symmetrizations of F converge in symmetric difference distance to the ball F^*.*

Recently Bianchi, Klain, Lutwak, Yang and Zhang (in [BKLYZ]) and, independently, Gronchi, proved that density alone of the sequence of directions $\{u_n\}$ in which the Steiner symmetrizations are taken is not sufficient for the convergence. In contrast Klain proved that a sequence of Steiner symmetrizations that uses only finitely many distinct directions always converges (but not necessarily to K^*).

In a recent paper we (Bianchi, Burchard, Gronchi and V.), addressed several questions that were raised in these papers. Our main result says that many of the known non-converging examples do converge if the Steiner symmetrizations are followed by suitable rotations. Such behavior of the sequence is called *convergence in shape*. This confirms a conjecture posed in [BKLYZ]. The limit is, in general, not an ellipsoid (or a convex set) unless the sequence starts from an ellipsoid (or a convex set, respectively).