Workshop on Topology of Wild Spaces and Fractals

July 4 – 8, 2011, Strobl (Austria)

Program, Abstracts, and List of Participants

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Program

Monday, 4th July	
8:00 – 8:50	*Breakfest*
8:50 – 9:00	Opening
9:00 – 9:40	Jerzy Dydak Covering maps for locally path-connected spaces
9:40 – 10:20	Curt Kent Local homotopy properties of asymptotic cones
10:20 – 10:40	*Coffee break*
10:40 – 11:20	Paul Fabel On quasitopological homotopy groups
11:20 – 12:00	Petar Pavešić Fibrations with unique path lifting property
12:00 – 13:00	*Lunch*
14:30 – 15:10	Conrad Plaut The $\mathbb{R}$-tree is the mother of all geodesic spaces
15:10 – 15:30	*Coffee break*
15:30 – 16:10	Ziga Virk A homotopically Hausdorff space which does not admit the universal covering space
16:10 – 16:50	Andreas Zastrow The Sombrero-space and the lessons that could be learnt from it
18:00 – 19:00	*Dinner*

Tuesday, 5th July	
8:00 – 9:00	*Breakfest*
9:00 – 9:40	Benoît Loridant Topology of a class of substitution tiles
9:40 – 10:20	Valérie Berthé Fractals and S-adic systems
10:20 – 10:40	*Coffee break*
10:40 – 11:20	Tai-Man Tang The structure of planar integral self-affine tiles with consecutive collinear digit set
11:20 – 12:00	Ligia Loretta Cristea On connectedness properties and distances between points of certain classes of Sierpiński carpets
12:00 – 13:00	*Lunch*
14:30 – 15:10	Mark Meilstrup Archipelago groups
15:10 – 15:30	*Coffee break*
15:30 – 16:10	Hanspeter Fischer Generalized Cayley graphs for fundamental groups of one-dimensional spaces
16:10 – 16:50	Wolfram Hojka One-ended fractals
18:00 – 19:00	*Dinner*

Wednesday, 6th July	
8:00 – 9:00	*Breakfest*
9:00 – 18:00	Free time for discussion, hiking, swimming, playing music, etc.
12:00 – 13:00	*Lunch (optional)*
18:00 – 19:00	*Dinner*

Thursday, 7th July	
8:00 – 9:00	*Breakfest*
9:00 – 9:40	Alexander Dranishnikov Cohomology of the Higson compactification
9:40 – 10:20	Neza Mramor-Kosta The cohomology of locally free groups
10:20 – 10:40	*Coffee break*
10:40 – 11:20	Agelos Georgakopoulos A new homology for infinite graphs and metric continua
11:20 – 12:00	Umed H. Karimov On cohomology manifolds which are not homology locally connected
12:00 – 13:00	*Lunch*
14:30 – 15:10	Katsuya Eda Group theoretic properties for wild algebraic topology
15:10 – 15:30	*Coffee break*
15:30 – 16:10	Gerhard Dorfer On the fundamental group of one-dimensional spaces
18:00 – 19:00	*Dinner*

Friday, 8th July	
8:00 – 9:00	*Breakfest*
9:00 – 9:40	Sam Corson Atomic weak presentations
9:40 – 10:20	Jörg Thuswaldner Topological properties of self-affine tiles
10:20 – 10:40	*Coffee break*
10:40 – 11:20	Greg Conner Some interesting open problems in low-dimensional wild topology
11:20 – 11:30	Closing
12:00 – 13:00	*Lunch*

Abstracts

Valérie Berthé

Tuesday, 9:40–10:20

Fractals and S-adic systems

It is well-known how to associate fractals with algebraic (and more precisely Pisot) parameters, both in the framework of numeration systems and of substitutive symbolic dynamical systems, in order to capture the discrete part of their spectrum. It is more delicate to associate such fractals to nonalgebraic parameters. To this end, substitutive symbolic dynamical systems or Pisot beta-numerations can be extended to linearly recurrent systems and more generally to so-called S-adic systems. These latter systems are obtained by iterating not only one substitution, but a finite number of them. The hierarchies thus produced have not necessarily the same structure at each level, but there are only finitely many possible structures. Our aim here is under suitable convergence properties that play the role of the Pisot assumption to define fractals with tiling properties in this framework. (Joint work with W. Steiner and J. Thuswaldner.)

Greg Conner

Friday, 10:40–11:20

Some interesting open problems in low-dimensional wild topology

In this talk I will discuss a number of open problems which I find highly interesting but difficult. I will make reference to a number of new results related to the open problems on which my coauthors will speak. For instance, one of the problems is whether the archipelago and the Griffiths space have the same fundamental group. We've made some interesting progress showing that many spaces have the same fundamental group as the archipelago, but the Griffiths space isn't one of these.

Sam Corson

Friday, 9:00–9:40

Atomic weak presentations

In this talk I discuss an application of certain groups which I call atomic weak presentations (elsewhere in the literature such structures are called omega groups) in homotopy theory. Such structures allow for restricted infinite multiplication, as in the fundamental group of the Hawaiian Earring, and allow for succinct descriptions of certain topological phenomena.

Ligia Loretta Cristea

Tuesday, 11:20–12:00

On connectedness properties and distances between points of certain classes of Sierpiński carpets

Sierpiński carpets are self-similar fractals in the plane that originate from the well-known Sierpiński carpet. They are constructed in the following way: start with the unit square, divide it into $n \times n$ congruent smaller subsquares and cut out m of them, corresponding to a given $n \times n$ pattern (called the generator of the Sierpiński carpet). By repeating this construction step with all the remaining subsquares ad infinitum, the resulting object is a fractal of Hausdorff and box-counting dimension $\frac{\log(n^2-m)}{\log(n)}$, called a *Sierpiński carpet*. At each step of the iterative construction the corresponding squares are deleted, together with their boundary, and then the closure (with respect to the topology induced by the Euclidean metric in the plane) is taken.

In this talk we present results on (dis)connectedness properties of different classes of Sierpiński carpets and generalisations thereof: limit net sets and generalised Sierpiński carpets. *Limit net sets* are in a certain sense "well-distributed" fractals, a special class of carpets. *Generalised Sierpiński carpets* are planar sets that are defined by means of sequences of patterns. Furthermore, we introduce and study *labyrinth fractals*. These are self-similar dendrites. We show that under certain conditions labyrinth fractals have the property that the distance between any two distinct points of the fractal is infinite.

(The results on labyrinth fractals and generalised Sierpiński carpets stem from joint work with Bertran Steinsky. Our research is supported by FWF-Project P20412-N18.)

Gerhard Dorfer

Thursday, 15:30–16:10

On the fundamental group of one-dimenisonal spaces

Let X be a compact metrizable one-dimensional connected space. In our talk we describe the fundamental group of X as subgroup of a projective limit of words. This description is used to give a new proof of a result due to Eda on continuity properties of homomorphisms from the fundamental group of the Hawaiian earring to X.

(This is joint work with J. Thuswaldner and R. Winkler)

Alexander Dranishnikov

Thursday, 9:00–9:40

Cohomology of the Higson compactification

The Higson compactification of a metric space is defined by means of the algebra of functions with the gradient tending to zero at infinity. It is a very wild space that resembles the Stone-Čech compactification. An acyclicity of the Higson compactification of the universal covering of a classifying space of a finitely presented group would imply many famous conjectures for that group. We will discuss the acyclicity problem of the Higson compactification of the Euclidean space and of the hyperbolic space.

Jerzy Dydak

Monday, 9:00–9:40

Covering maps for locally path-connected spaces

We define Peano covering maps and prove basic properties analogous to classical covers. Their domain is always locally path-connected but the range may be an arbitrary topological space. One of characterizations of Peano covering maps is via the uniqueness of homotopy lifting property for all locally path-connected spaces.

Regular Peano covering maps over path-connected spaces are shown to be identical with generalized regular covering maps introduced by Fischer and Zastrow. If X is path-connected, then every Peano covering map is equivalent to the projection $\widetilde{X}/H \to X$, where H is a subgroup of the fundamental group of X and $\widetilde{X}$ equipped with the basic topology. The projection $\widetilde{X}/H \to X$ is a Peano covering map if and only if it has the unique path lifting property. We define a new topology on $\widetilde{X}$ for which one has a characterization of $\widetilde{X}/H \to X$ having the unique path lifting property if H is a normal subgroup of $\pi_1(X)$. Namely, H must be closed in $\pi_1(X)$. Such groups include $\pi(\mathcal{U}, x_0)$ ($\mathcal{U}$ being an open cover of X) and the kernel of the natural homomorphism from the fundamental group to the Cech fundamental group.

(This is joint work with N. Brodskiy, B. Labuz, A. Mitra.)

Katsuya Eda

Thursday, 14:30–15:10

Group theoretic properties for wild algebraic topology

In this talk I'll talk about group theoretic properties which are related to algebraic topology of wild spaces, i.e. nonsemi-locally simply connected spaces. Specker phenomenon, slender groups, n-slender groups, contorsionfree groups, algebraical compact groups, the p-adic integer groups and divisible groups will be mentioned.

Paul Fabel

Monday, 10:40–11:20

On quasitopological homotopy groups

The familar nth homotopy group G of a space X becomes a quasitopological group when endowed with the natural quotient topology inherited from the space of based maps of the n-sphere into X. The space G is an invariant of the homotopy type of X, and has the capacity to distinguish X from Y when standard application of shape theory fails to do so.

Building on work of the author (and independent work of Jeremy Brazas) we will discuss examples in which G fails to be a topological group and theorems which ensure G is in fact a topological group.

Hanspeter Fischer

Tuesday, 15:30–16:10

Generalized Cayley graphs for fundamental groups of one-dimensional spaces

Fundamental groups of general one-dimensional Peano continua, such as the Menger curve, are notoriously difficult to analyze. They are neither free nor countable when small essential loops accumulate in the underlying space. Yet, every finitely generated subgroup of the fundamental group of a one-dimensional separable metric space is free and the homotopy class of every loop contains an essentially unique shortest representative. In light of these and related results, J.W. Cannon and G.R. Conner have asked whether every one-dimensional path-connected compact metric space admits a tree-like object that might be considered the topological Cayley graph of its fundamental group. We answer this question in the positive. Using a generalized universal covering space, we give a combinatorial description of an $\mathbb{R}$-tree which, to the extent possible, functions like a Cayley graph for such a group.

(This is joint research with Andreas Zastrow of the University of Gdańsk, Poland.)

Agelos Georgakopoulos

Thursday, 10:40–11:20

A new homology for infinite graphs and metric continua

We generalise a fundamental graph-theoretical fact, stating that every element of the cycle space of a graph is a sum of edge-disjoint cycles, to arbitrary continua. To achieve this we replace graph cycles by topological circles, and replace the cycle space of a graph by a new homology group for continua which is a quotient of the first singular homology group H_1. This homology seems to be particularly apt for studying spaces with infinitely generated H_1, e.g. infinite graphs or fractals.

Wolfram Hojka

Tuesday, 16:10–16:50

One-ended fractals

We present a new approach to determine topological properties of fractals. If the semigroup generated by an iterated function system is one-ended, the associated fractal has to be connected. We then apply this technique to some examples, and ask the question in what classes of transformations (e.g. affine, piecewise linear) the construction can be reversed.

(This is joint research with Greg Conner.)

Umed H. Karimov

Thursday, 11:20–12:00

On cohomology manifolds which are not homology locally connected

G. Bredon constructed a 2-dimensional compact cohomology manifold which is not homologically locally connected with respect to singular homology (not an HLC space) [2]. This space is of course not metrizable because any metrizable 2-dimensional cohomology manifold is a usual manifold and therefore locally contractible. In the paper [3] a 3-dimensional compact metrizable cohomology manifold was constructed which is not an HLC space. We call this spaces almost Euclidean manifolds. In the paper [1] the construction of van Kampen of compact metrizable generalised manifold was given. The van Kampen manifold is a cohomology manifold which is not an HLC space. The main purpose of this report is to show that any almost Euclidean manifold is not homeomorphic to any van Kampen manifold.

References

[1] E. G. Begle, Locally connected spaces and generalized manifolds, Amer. J. Math. 64 (1942), 553–574.

[2] G. E. Bredon, Sheaf Theory, 2nd Ed., Graduate Texts in Math. 170, Springer, Berlin, 1997.

[3] U. H. Karimov, D. Repovš, Examples of cohomology manifolds which are not homologically locally connected, Topology Appl. 155 (2008), 1169–1174.

(This is joint work with D. Repovš.)

Curt Kent

Monday, 9:40–10:20

Local homotopy properties of asymptotic cones

An asymptotic cone of a group is a metric space which encodes the large scale geometry of a group. We will briefly define asymptotic cones and give some examples of cones with locally non-trivial topologies. In '91, Gromov asked what groups arise as fundamental groups of asymptotic cones of finitely generated groups. By examining the local fundamental groups of asymptotic cones of multiple HNN extensions of free groups, we will show that an asymptotic cone of a group from this class has fundamental group which is trivial or uncountable. As a corollary of the proof, we see that these cones are locally simply connected if and only if they are semi-locally simply connected.

Benoît Loridant

Tuesday, 9:00–9:40

Topology of a class of substitution tiles

Consider the Tribonacci substitution $\sigma(1) = 12, \sigma(2) = 13, \sigma(3) = 1$. In 1982, Gérard Rauzy realized geometrically the uniquely ergodic dynamical system generated by this substitution. It is measure theoretically conjugate to an exchange of domains in a plane compact set, the Rauzy fractal. This talk is devoted to a family of substitutions having similar properties as the Tribonacci substitution:

$$\sigma(1) = \underbrace{1\ldots1}_{a}2 \qquad \sigma(2) = \underbrace{1\ldots1}_{b}3 \qquad \sigma(3) = 1$$

($a \geq b \geq 1$). We will study the topology of the associated fractals: boundary parametrization, homeomorphy to the closed disk, etc.

Mark Meilstrup

Tuesday, 14:30–15:10

Archipelago groups

The harmonic archipelago is a noncompact space that is obtained by adjoining a sequence of 'tall' discs between consecutive loops of the Hawaiian earring. We discuss the fundamental group of the Harmonic archipelago, and generalize this to a construction for an arbitrary sequence of groups, which we call an archipelago group. We discuss properties of the resulting construction, including an interesting almost-uniqueness result.

Neza Mramor-Kosta

Thursday, 9:40–10:20

The cohomology of locally free groups

We will present some cohomological properties of locally free non free groups which arise from a representation of such groups as limits of free groups. Our motivating example will be the fundamental group of a grope.

Petar Pavešić

Monday, 11:20–12:00

Fibrations with unique path lifting property

Fibrations that admit unique path liftings are a natural extension of the concept of coverings. They appear in a variety of situations, e.g. when taking inverse limits of coverings. We are going to discuss some interesting examples and describe the relation between the fundamental group of the base space, the so-called shape-kernel group and the group of deck transformations of the fibration.

(This is joint work with G. Conner.)

Conrad Plaut

Monday, 14:30–15:10

The $\mathbb{R}$-tree is the mother of all geodesic spaces

In joint work with V. N. Berestovskii, we show that every geodesic space is the metric quotient of an $\mathbb{R}$-tree, called the covering $\mathbb{R}$-tree, via a free isometric action of a locally free group. The quotient mapping is a kind of generalized covering map called a URL-map (Unique Rectifiable Lifting), and the covering $\mathbb{R}$-tree is universal among URL-maps onto the space. In the case of certain well-known fractals and complete Riemannian manifolds of dimension at least two, the covering $\mathbb{R}$-tree is always the unique "universal" $\mathbb{R}$-tree with valency of cardinality the continuum at every point, and the isometry group of the action is not free and not a free product of surface groups and abelian groups–giving many new (negative) answers to a question of J. W. Morgan. If there is time, I'll discuss some other applications and preliminary results, including a new answer to a classical question about open, light, dimension increasing maps, and "designer fibrations" that can be produced as quotients of the covering $\mathbb{R}$-tree.

Tai-Man Tang

Tuesday, 10:40–11:20

The structure of planar integral self-affine tiles with consecutive collinear digit set

We investigate the topological structure of planar integral self-affine tiles $T(A, \mathcal{D})$, the solution to $T = \bigcup_{d\in\mathcal{D}}(A^{-1}1T + d)$ where $A \in M(2;\mathbb{Z})$ is an expanding integral matrix and $\mathcal{D}$ a consecutive collinear digit set. There are three cases: disklike tiles, non-disklike tiles with $\det A > 0$ and $\det A < 0$. We will present results indicating the following structures. In the second case, T has a hole, and hence infinitely many by a result of Luo and Thuswaldner (2006). In the third case, T is like a two-tail sequence of diminishing disks.

The structure of the tilings by such tiles is completely known. In Leung and Lau (2007), a necessary and sufficient condition for A to be dislike, and the structure of the tilings they generated, are given in terms of the characteristic polynomial of A. Deng, Jiang and Ngai (to appear) completely determine the structure of the tilings by the non-disklike T's. This work is an attempt to describe the topological structure of such tiles.

(This is joint work with Wei-Chao Guo of the Zhejiang University, People's Republic of China).

Jörg Thuswaldner

Friday, 9:40–10:20

Topological properties of self-affine tiles

In this talk we want to survey results on topological properties of self-affine sets. We start with results of Hata on connectivity as well as criteria on arclikeness. In a second part we discuss topological properties of self-affine subsets of the plane. Here the Jordan curve theorem and its consequences allows to give quite simple criteria for the homeomorphy to a disk. In the last part we deal with representations of self-affine fractals as metric projective limits. Together with methods from low dimensional topology such representations enable us to prove topological results on self-affine sets also in higher dimensions.

Ziga Virk

Monday, 15:30–16:10

A homotopically Hausdorff space which does not admit the universal covering space

In attempt to define the universal covering spaces in the category of spaces which are not semilocally simply connected, the separation property of being homotopically Hausdorff was defined. It is a weakening of semilocal simple connectedness and has turned out to be a necessary condition for the existence of the universal covering space. Well known examples of spaces violating this property include Griffiths' space and the Harmonic Archipelago. In this talk we will present a homotopically Hausdorff space which does not admit the universal covering space thus proving that the two properties are not equivalent.

(This is joint work with Andreas Zastrow.)

Andreas Zastrow

Monday, 16:10–16:50

The Sombrero-space and the lessons that could be learnt from it

The essential part of the so-called Sombrero-space is a rotated topologist's sine curve, rotated around its tame end. Arcs have been added to this space to make it a Peano continuum. It already appeared in published joint literature with Conner, Meilstrup, Repovš & Željko and with Fischer, Repovš & Virk. It turned out that this space has an interesting combination of properties of homotopic Hausdorffness, on the one hand not containing a direct obstruction for the existence of generalized covering spaces, but on the other hand making it very difficult to construct them. The talk will first briefly explain and review the known properties of this space, and then describe an approach to understand the essential properties of its fundamental group. This approach finally allows to confirm that this space has indeed a generalized universal covering space. Hence the analysis of this space has also lead to the discoverage of a weaker criterium for the existence of generalized universal covering spaces than in the prior joint research with Hanspeter Fischer.

List of participants

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