CSASC 2011

Minisymposium on
Categorical Algebra, Homotopy Theory, and Applications

This minisymposium encompasses a broad area where algebraic and category-theoretical methods are combined, aiming at applications to algebraic topology, algebraic geometry and representation theory. Topics include rings and modules, representable functors, accessible categories, triangulated categories, homotopical algebra, higher-order homotopy structures, and other related subjects.

ORGANIZERS

Carles Casacuberta, Universitat de Barcelona
Jiří Rosický, Masarykova univerzita, Brno
Jan Trlifaj, Univerzita Karlova, Praha

SCHEDULE

Monday, September 26, 15.00 – 18.00, SE 1.2

Michael Makkai, Brno	*Kan complexes and face structures*
Fernando Muro, Sevilla	*Moduli spaces of differential graded algebra structures*
Jan Šťovíček, Praha	*Generating the bounded derived category and perfect ghosts*
Oriol Raventós, Brno	*Adams representability in triangulated categories*
Javier Gutiérrez, Barcelona	*Generalized Ohkawa's theorem*
David Pospíšil, Praha	*(Co)tilting classes over commutative noetherian rings*

Tuesday, September 27, 10.30 – 12.30, SE 1.2

Imma Gálvez, Barcelona	*Generalized syzygies for commutative Koszul algebras*
Pavel Příhoda, Praha	*Projective modules over universal enveloping algebras*
Lukáš Vokřínek, Brno	*When is the colimit functor homotopy invariant?*
George Raptis, Osnabrück	*The stable model categories of modules over the Frobenius rings* $\mathbf{F}_p[\varepsilon]/\varepsilon^2$ *and* $\mathbf{Z}/p^2$

Tuesday, September 27, 14.45 – 16.45, SE 1.2

Beatriz Rodríguez, Madrid	*A characterization of homotopically cocomplete categories*
Ilias Amrani, Brno	*Infinity categories*
Pavel Růžička, Praha	*Computing* $V(R)$ *of regular rings*
Andrew Tonks, London	*Crossed complex resolutions of group extensions*

ABSTRACTS OF TALKS

Ilias Amrani, Masarykova univerzita, Brno
Infinity categories

We compare and explain from a homotopical point of view different models for the notion of infinity categories (simplicial sets, simplicial categories) constructed by A. Joyal and J. Bergner. We construct a new model for infinity categories, namely the model category of topological categories $\mathbf{Cat}_{\mathbf{Top}}$.

If time permits, we will discuss some potential consequences of this new model.

Imma Gálvez Carrillo, Universitat Politècnica de Catalunya, Terrassa
Generalized syzygies for commutative Koszul algebras

I will report on work in progress with Vassily Gorbounov (Aberdeen), Zain Shaikh (Cologne) and Andrew Tonks (Londonmet) on the relations between Koszul duality and cohomology. We will consider the generalized syzygies of a commutative finitely generated Koszul algebra and we will prove that they agree with the cohomology of the Lie ideal $L_{\geq 3}$ of the graded Lie superalgebra A such that $A^{!} \approx U(L)$. This generalizes results by Movshev and Schwarz, and by Gorodentsev, Khoroshkin and Rudakov. The proof uses homological algebra techniques, in particular the homotopy perturbation lemma. Also, we conjecture that $L_{\geq 3}$ will be a free algebra and proving it would help to give support to an old conjecture by Avramov.

Javier J. Gutiérrez, Universitat de Barcelona
Generalized Ohkawa's theorem

The classical Ohkawa theorem states that in the homotopy category of spectra there is only a set of Bousfield classes. This fact has been studied and generalized to other triangulated categories by Neeman, Iyengar–Krause and Dwyer–Palmieri. In this talk we will present a generalization of Ohkawa's theorem in the context of combinatorial model categories. This is a joint work with C. Casacuberta and J. Rosický.

Michael Makkai, Masarykova univerzita, Brno
Kan complexes and face structures

Let Λ denote the subcategory of Δ (familiar from simplicial sets) whose objects are the same as those of Δ, but whose arrows are the injective maps (face operators) only. A "face structure" is a presheaf on Λ. Every simplicial set has an underlying face structure. The talk is about results around the fact that the full simplicial structure on a Kan complex is merely a property of the underlying face structure of the complex, similarly to being an elementary topos is merely a property of the underlying category. The precise results include a characterization in FOLDS (First Order Logic with Dependent Sorts) of the homotopy invariant first-order language of Kan complexes. There are generalizations to quasi categories and Θ categories (A. Joyal). Some, but not all, of the present subject I talked about in Durham, England, in 2009.

Fernando Muro, Universidad de Sevilla
Moduli spaces of differential graded algebra structures

In this talk I will present a comparison between the spaces of unital and non-unital differential graded algebra structures on a given chain complex. This sheds some light from an operadic point of view on various normalization results in the literature.

David Pospíšil, Univerzita Karlova, Praha
(Co)tilting classes over commutative noetherian rings

In the talk, I will discuss a classification of tilting and cotilting classes over commutative noetherian rings, recently obtained jointly with Lidia Angeleri, Jan Šťovíček and Jan Trlifaj. The classification is in terms of certain subsets of the Zariski spectrum of the corresponding ring.

Pavel Příhoda, Univerzita Karlova, Praha
Projective modules over universal enveloping algebras

I will survey results on projective modules over universal enveloping algebras of finite dimensional Lie algebras over a field of characteristic zero. I will focus on the relation between direct sum decompositions of infinitely generated projective modules and the structure of the Lie algebra.

George Raptis, Universität Osnabrück
The stable model categories of modules over the Frobenius rings $\mathbf{F}_p[\varepsilon]/\varepsilon^2$ *and* $\mathbf{Z}/p^2$

In this talk I will survey some results about the homotopy theories of the title as they were studied by Schlichting, Dugger–Shipley and in joint work with Muro. This is a fascinating case study for homotopy theory and K-theory and has led to some interesting conclusions. Time permitting, the following topics will be discussed:

(a) The associated homotopy categories are triangulated-equivalent but the model categories are not Quillen equivalent. This connects to the problem of the rigidity of a stable homotopy theory which is nicely analysed using the methods of derived Morita theory.

(b) The associated triangulated derivators cannot be equivalent; however, they agree on the domain of finite ordinals. This connects to the general question of the comparison between model categories and their associated derivators.

(c) They have different Waldhausen K-theory, which can be regarded as a subtle invariant of homotopy theories, but the same derivator K-theory. This leads to some negative conclusions for derivator K-theory.

Oriol Raventós, Masarykova univerzita, Brno
Adams representability in triangulated categories

We will discuss recent results about the representability of cohomological functors defined from a subcategory of compact objects (with respect to a fixed cardinal) of a well-generated triangulated category.

Given a triangulated category T and a regular cardinal κ, we say that T satisfies Adams representability with respect to κ if every cohomological functor from the category of κ-compact objects in T to the category of abelian groups that sends coproducts of less than κ objects to products is the restriction of a representable functor $\mathrm{Hom}(-, X)$ with X an object in T.

If κ is countable, Adams representability is a classical result proved by Adams in the stable homotopy category, who inferred the representability of homology theories, and it was later generalized by Neeman to a broader family of triangulated categories.

We will impose conditions on a triangulated category ensuring that it satisfies Adams representability with respect to an arbitrary regular cardinal κ. We will focus on concrete examples in the case when κ is $\aleph_1$.

Beatriz Rodríguez González, ICMAT, Madrid
A characterization of homotopically cocomplete categories

In this talk we review different notions of homotopy colimits and homotopy cocomplete categories appearing in the literature, and describe some relations between them. In addition, for categories with exact coproducts we give a characterization of homotopical cocompleteness based on the existence of "homotopy coequalizers", interpreted as good homotopy colimits for diagrams of simplicial shape. This characterization might be understood as a homotopical version of the classical result stating that all colimits can be computed using coproducts and coequalizers.

Pavel Růžička, Univerzita Karlova, Praha
Computing $V(R)$ of regular rings

Given a ring R, $V(R)$ stands for the commutative monoid of isomorphism classes of finitely generated projective right R-modules. A monoid realized as $V(R)$ of a regular ring is easily seen to be conical and to satisfy the refinement property. These properties, however, do not characterize all realizable commutative monoids due to Wehrung's counter-example. The size of the counter-example is at least $\aleph_2$; thus it leaves open the question whether all conical refinement monoids of smaller size (in particular all countable ones) are realizable. This is often quoted as the fundamental problem posed by K. R. Goodearl. We will discuss some ways how to attack this problem and present some examples of non-trivial representable monoids.

Jan Šťovíček, Univerzita Karlova, Praha
Generating the bounded derived category and perfect ghosts

This is an account on joint work with S. Oppermann. We show, for a wide class of abelian categories studied in representation theory and algebraic geometry, that there are no non-trivial thick subcategories of the bounded derived category containing all perfect complexes and having finite Rouquier's dimension. This holds in particular for the category of finitely generated modules over an Artin algebra, or for the category of coherent sheaves over an affine or projective scheme.

Andrew Tonks, London Metropolitan University
Crossed complex resolutions of group extensions

An old paper of C. T. C. Wall shows, with a simple spectral sequence argument, that given free chain resolutions for groups *H* and *K* one may construct a resolution for any group extension *G* of *H* by *K*. More recently, Brown, Ellis and others have attempted, with some degree of success, to lift this construction from the category of chain complexes to that of crossed complexes or of spaces. This non-abelian situation is considerably harder; one knows, for example, that there is no homological perturbation theory for crossed complexes. In this talk we will give an overview of the problem and present some new results obtained in collaboration with O. Gill.

Lukáš Vokřínek, Masarykova univerzita, Brno
When is the colimit functor homotopy invariant?

It is well known that for a (cofibrantly generated) model category **M** the colimit functor from [*I*, **M**] to **M** preserves weak equivalences between diagrams that are cofibrant in the projective model structure on the diagram category [*I*, **M**]. Some special cases are known where such homotopy invariance holds in greater generality; for example, in the case of pushouts. We generalize this to other indexing categories *I*.

Our main application is a construction of a well-behaved cofibrant replacement functor in the category $\mathbf{V\text{-}Cat}_{O}$ ot categories enriched in a monoidal model category **V** with a fixed set *O* of objects. We hope that it will serve to extend the theory of homotopy coherent diagrams (well known if **V** is the category of simplicial sets) to arbitrary bases.